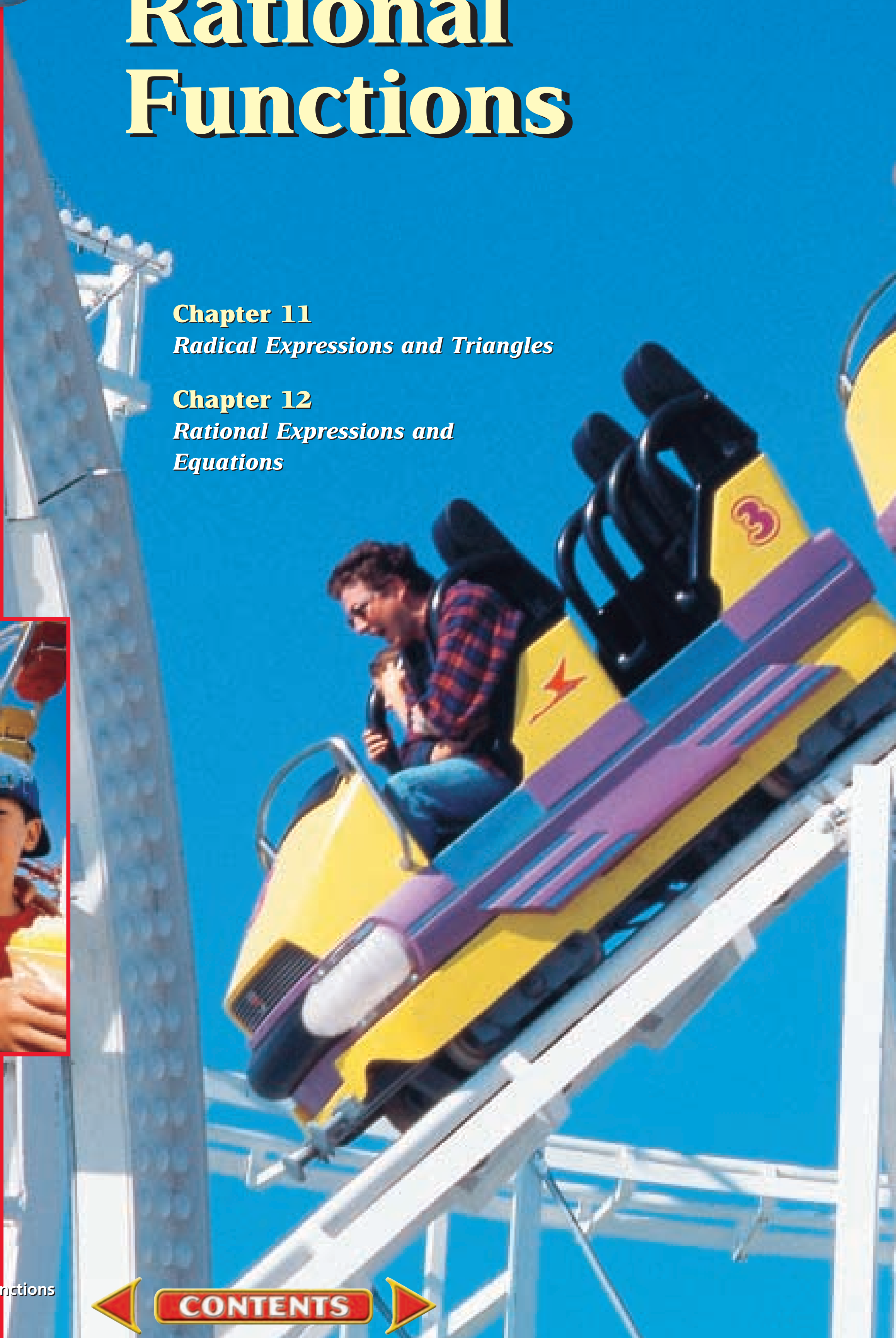
UNIT 4

Radical and Rational Functions

Nonlinear functions such as radical and rational functions can be used to model real-world situations such as the speed of a roller coaster. In this unit, you will learn about radical and rational functions.



WebQuest Internet Project

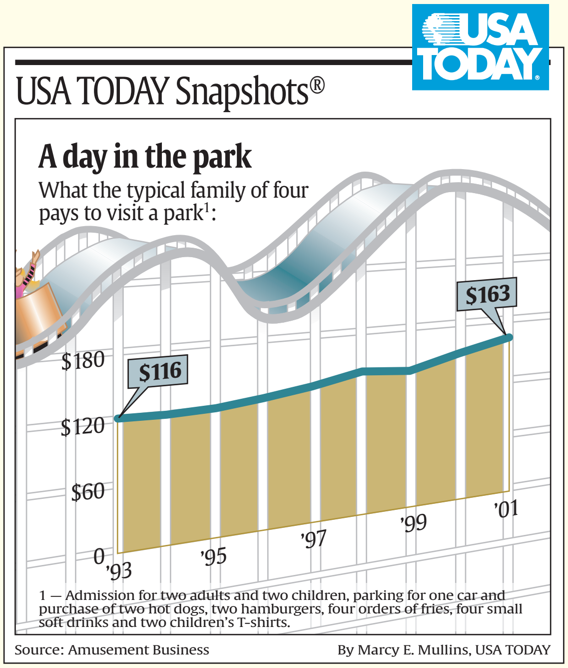
Building the Best Roller Coaster

Each year, amusement park owners compete to earn part of the billions of dollars Americans spend at amusement parks. Often the parks draw customers with new taller and faster roller coasters. In this project, you will explore how radical and rational functions are related to buying and building a new roller coaster.

Log on to www.algebra1.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 4.

Lesson	11-1	12-2
Page	590	652



Chapter 11

Radical Expressions and Triangles

What You'll Learn

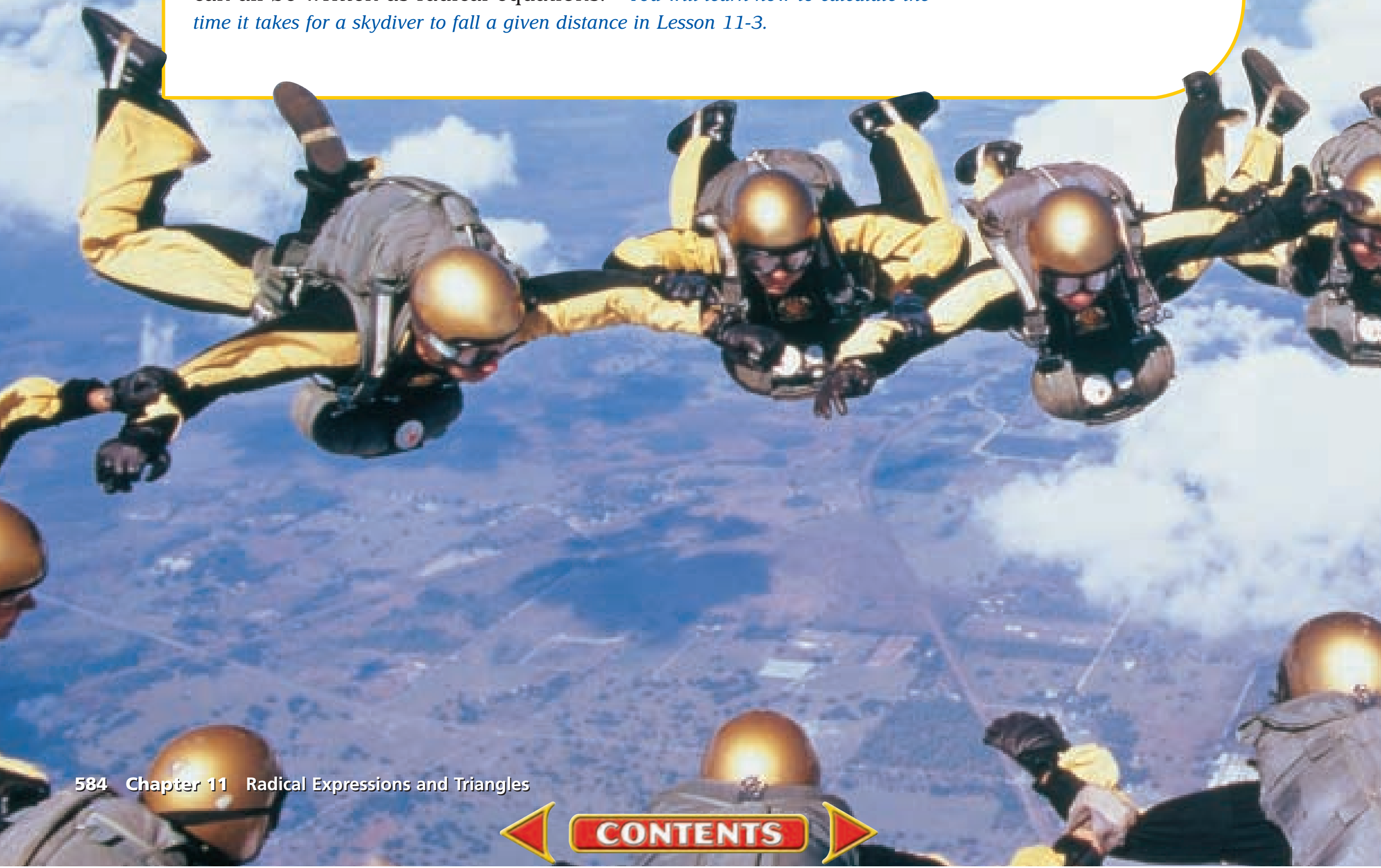
- **Lessons 11-1 and 11-2** Simplify and perform operations with radical expressions.
- **Lesson 11-3** Solve radical equations.
- **Lessons 11-4 and 11-5** Use the Pythagorean Theorem and Distance Formula.
- **Lessons 11-6 and 11-7** Use similar triangles and trigonometric ratios.

Key Vocabulary

- radical expression (p. 586)
- radical equation (p. 598)
- Pythagorean Theorem (p. 605)
- Distance Formula (p. 611)
- trigonometric ratios (p. 623)

Why It's Important

Physics problems are among the many applications of radical equations. Formulas that contain the value for the acceleration due to gravity, such as free-fall times, escape velocities, and the speeds of roller coasters, can all be written as radical equations. *You will learn how to calculate the time it takes for a skydiver to fall a given distance in Lesson 11-3.*



Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 11.

For Lessons 11-1 and 11-4 — Find Square Roots

Find each square root. If necessary, round to the nearest hundredth.
(For review, see Lesson 2-7.)

1. $\sqrt{25}$ **2.** $\sqrt{80}$ **3.** $\sqrt{56}$ **4.** $\sqrt{324}$

For Lesson 11-2 — Combine Like Terms

Simplify each expression. *(For review, see Lesson 1-6.)*

5. $3a + 7b - 2a$
6. $14x - 6y + 2y$
7. $(10c - 5d) + (6c + 5d)$
8. $(21m + 15n) - (9n - 4m)$

For Lesson 11-3 — Solve Quadratic Equations

Solve each equation. *(For review, see Lesson 9-3.)*

9. $x(x - 5) = 0$
10. $x^2 + 10x + 24 = 0$
11. $x^2 - 6x - 27 = 0$
12. $2x^2 + x + 1 = 2$

For Lesson 11-6 — Proportions

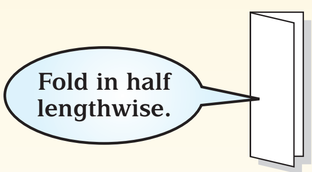
Use cross products to determine whether each pair of ratios forms a proportion. Write *yes* or *no*. *(For review, see Lesson 3-6.)*

13. $\frac{2}{3}, \frac{8}{12}$ **14.** $\frac{4}{5}, \frac{16}{25}$ **15.** $\frac{8}{10}, \frac{12}{16}$ **16.** $\frac{6}{30}, \frac{3}{15}$

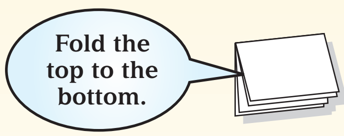
FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about radical expressions and equations. Begin with a sheet of plain $8\frac{1}{2}$" by 11" paper.

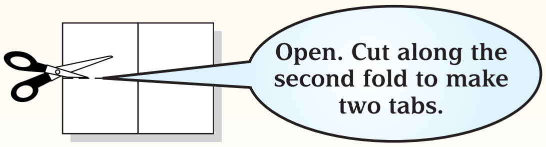
Step 1 Fold in Half



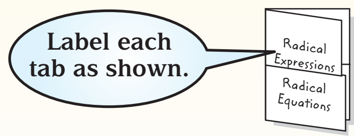
Step 2 Fold Again



Step 3 Cut



Step 4 Label



Reading and Writing As you read and study the chapter, write notes and examples for each lesson under each tab.

11-1 Simplifying Radical Expressions

What You'll Learn

- Simplify radical expressions using the Product Property of Square Roots.
- Simplify radical expressions using the Quotient Property of Square Roots.

How are radical expressions used in space exploration?

A spacecraft leaving Earth must have a velocity of at least 11.2 kilometers per second (25,000 miles per hour) to enter into orbit. This velocity is called the *escape velocity*. The escape velocity of an object is given by the radical expression $\sqrt{\frac{2GM}{R}}$, where G is the gravitational constant, M is the mass of the planet or star, and R is the radius of the planet or star. Once values are substituted for the variables, the formula can be simplified.

Vocabulary

- radical expression
- radicand
- rationalizing the denominator
- conjugate

PRODUCT PROPERTY OF SQUARE ROOTS A **radical expression** is an expression that contains a square root. A **radicand**, the expression under the radical sign, is in simplest form if it contains no perfect square factors other than 1. The following property can be used to simplify square roots.

Key Concept — *Product Property of Square Roots*

- **Words** For any numbers a and b, where $a \geq 0$ and $b \geq 0$, the square root of a product is equal to the product of each square root.
- **Symbols** $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$
- **Example** $\sqrt{4 \cdot 25} = \sqrt{4} \cdot \sqrt{25}$

The Product Property of Square Roots and prime factorization can be used to simplify radical expressions in which the radicand is not a perfect square.

Example 1 *Simplify Square Roots*

Simplify.

a. $\sqrt{12}$

$\sqrt{12} = \sqrt{2 \cdot 2 \cdot 3}$ Prime factorization of 12

$= \sqrt{2^2} \cdot \sqrt{3}$ Product Property of Square Roots

$= 2\sqrt{3}$ Simplify.

b. $\sqrt{90}$

$\sqrt{90} = \sqrt{2 \cdot 3 \cdot 3 \cdot 5}$ Prime factorization of 90

$= \sqrt{3^2} \cdot \sqrt{2 \cdot 5}$ Product Property of Square Roots

$= 3\sqrt{10}$ Simplify.

Study Tip

Reading Math
$2\sqrt{3}$ is read *two times the square root of 3* or *two radical three.*

The Product Property can also be used to multiply square roots.

Study Tip

Alternative Method
To find $\sqrt{3} \cdot \sqrt{15}$, you could multiply first and then use the prime factorization.

$$\begin{aligned}\sqrt{3} \cdot \sqrt{15} &= \sqrt{45} \\ &= \sqrt{3^2} \cdot \sqrt{5} \\ &= 3\sqrt{5}\end{aligned}$$

Example 2 Multiply Square Roots

Find $\sqrt{3} \cdot \sqrt{15}$.

$$\begin{aligned}\sqrt{3} \cdot \sqrt{15} &= \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{5} && \text{Product Property of Square Roots} \\ &= \sqrt{3^2} \cdot \sqrt{5} && \text{Product Property} \\ &= 3\sqrt{5} && \text{Simplify.}\end{aligned}$$

When finding the principal square root of an expression containing variables, be sure that the result is not negative. Consider the expression $\sqrt{x^2}$. It may seem that $\sqrt{x^2} = x$. Let's look at $x = -2$.

$$\begin{aligned}\sqrt{x^2} &\stackrel{?}{=} x \\ \sqrt{(-2)^2} &\stackrel{?}{=} -2 && \text{Replace } x \text{ with } -2. \\ \sqrt{4} &\stackrel{?}{=} -2 && (-2)^2 = 4 \\ 2 &\neq -2 && \sqrt{4} = 2\end{aligned}$$

For radical expressions where the exponent of the variable inside the radical is *even* and the resulting simplified exponent is odd, you must use absolute value to ensure nonnegative results.

$\sqrt{x^2} = |x|$ $\quad \sqrt{x^3} = x\sqrt{x}$ $\quad \sqrt{x^4} = x^2$ $\quad \sqrt{x^5} = x^2\sqrt{x}$ $\quad \sqrt{x^6} = |x^3|$

Example 3 Simplify a Square Root with Variables

Simplify $\sqrt{40x^4y^5z^3}$.

$$\begin{aligned}\sqrt{40x^4y^5z^3} &= \sqrt{2^3 \cdot 5 \cdot x^4 \cdot y^5 \cdot z^3} && \text{Prime factorization} \\ &= \sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{5} \cdot \sqrt{x^4} \cdot \sqrt{y^4} \cdot \sqrt{y} \cdot \sqrt{z^2} \cdot \sqrt{z} && \text{Product Property} \\ &= 2 \cdot \sqrt{2} \cdot \sqrt{5} \cdot x^2 \cdot y^2 \cdot \sqrt{y} \cdot |z| \cdot \sqrt{z} && \text{Simplify.} \\ &= 2x^2y^2|z|\sqrt{10yz} && \text{The absolute value of } z \text{ ensures a nonnegative result.}\end{aligned}$$

QUOTIENT PROPERTY OF SQUARE ROOTS You can divide square roots and simplify radical expressions that involve division by using the Quotient Property of Square Roots.

Key Concept ***Quotient Property of Square Roots***

- **Words** For any numbers a and b, where $a \geq 0$ and $b > 0$, the square root of a quotient is equal to the quotient of each square root.
- **Symbols** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- **Example** $\sqrt{\frac{49}{4}} = \frac{\sqrt{49}}{\sqrt{4}}$

Study Tip

Look Back
To review the **Quadratic Formula**, see Lesson 10-4.

You can use the Quotient Property of Square Roots to derive the Quadratic Formula by solving the quadratic equation $ax^2 + bx + c = 0$.

$$\begin{aligned}ax^2 + bx + c &= 0 && \text{Original equation} \\ x^2 + \frac{b}{a}x + \frac{c}{a} &= 0 && \text{Divide each side by } a, a \neq 0.\end{aligned}$$

(continued on the next page)

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$ Subtract $\frac{c}{a}$ from each side.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$ Complete the square; $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$ Factor $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}$.

$$\left|x + \frac{b}{2a}\right| = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$ Take the square root of each side.

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$ Remove the absolute value symbols and insert $\pm$.

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$ Quotient Property of Square Roots

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$ $\sqrt{4a^2} = 2a$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Subtract $\frac{b}{2a}$ from each side.

Study Tip

Plus or Minus Symbol

The $\pm$ symbol is used with the radical expression since both square roots lead to solutions.

Thus, we have derived the Quadratic Formula.

A fraction containing radicals is in simplest form if no prime factors appear under the radical sign with an exponent greater than 1 and if no radicals are left in the denominator. **Rationalizing the denominator** of a radical expression is a method used to eliminate radicals from the denominator of a fraction.

Example 4 Rationalizing the Denominator

Simplify.

a. $\frac{\sqrt{10}}{\sqrt{3}}$

$$\frac{\sqrt{10}}{\sqrt{3}} = \frac{\sqrt{10}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$ Multiply by $\frac{\sqrt{3}}{\sqrt{3}}$.

$$= \frac{\sqrt{30}}{3}$$ Product Property of Square Roots

b. $\frac{\sqrt{7x}}{\sqrt{8}}$

$$\frac{\sqrt{7x}}{\sqrt{8}} = \frac{\sqrt{7x}}{\sqrt{2 \cdot 2 \cdot 2}}$$ Prime factorization

$$= \frac{\sqrt{7x}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$ Multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.

$$= \frac{\sqrt{14x}}{4}$$ Product Property of Square Roots

c. $\frac{\sqrt{2}}{\sqrt{6}}$

$$\frac{\sqrt{2}}{\sqrt{6}} = \frac{\sqrt{2}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$$ Multiply by $\frac{\sqrt{6}}{\sqrt{6}}$.

$$= \frac{\sqrt{12}}{6}$$ Product Property of Square Roots

$$= \frac{\sqrt{2 \cdot 2 \cdot 3}}{6}$$ Prime factorization

$$= \frac{2\sqrt{3}}{6}$$ $\sqrt{2^2} = 2$

$$= \frac{\sqrt{3}}{3}$$ Divide the numerator and denominator by 2.

Binomials of the form $p\sqrt{q} + r\sqrt{s}$ and $p\sqrt{q} - r\sqrt{s}$ are called **conjugates**. For example, $3 + \sqrt{2}$ and $3 - \sqrt{2}$ are conjugates. Conjugates are useful when simplifying radical expressions because if p, q, r, and s are rational numbers, their product is always a rational number with no radicals. Use the pattern $(a - b)(a + b) = a^2 - b^2$ to find their product.

$$(3 + \sqrt{2})(3 - \sqrt{2}) = 3^2 - (\sqrt{2})^2 \quad a = 3, b = \sqrt{2}$$

$$= 9 - 2 \text{ or } 7 \quad (\sqrt{2})^2 = \sqrt{2} \cdot \sqrt{2} \text{ or } 2$$

Example 5 Use Conjugates to Rationalize a Denominator

Simplify $\dfrac{2}{6 - \sqrt{3}}$.

$$\frac{2}{6 - \sqrt{3}} = \frac{2}{6 - \sqrt{3}} \cdot \frac{6 + \sqrt{3}}{6 + \sqrt{3}} \quad \frac{6 + \sqrt{3}}{6 + \sqrt{3}} = 1$$

$$= \frac{2(6 + \sqrt{3})}{6^2 - (\sqrt{3})^2} \quad (a - b)(a + b) = a^2 - b^2$$

$$= \frac{12 + 2\sqrt{3}}{36 - 3} \quad (\sqrt{3})^2 = 3$$

$$= \frac{12 + 2\sqrt{3}}{33} \quad \text{Simplify.}$$

When simplifying radical expressions, check the following conditions to determine if the expression is in simplest form.

Concept Summary — Simplest Radical Form

A radical expression is in simplest form when the following three conditions have been met.

1. No radicands have perfect square factors other than 1.
2. No radicands contain fractions.
3. No radicals appear in the denominator of a fraction.

Check for Understanding

Concept Check

1. **Explain** why absolute value is not necessary for $\sqrt{x^4} = x^2$.
2. **Show** that $\dfrac{1}{\sqrt{a}} = \dfrac{\sqrt{a}}{a}$ for $a > 0$.
3. **OPEN ENDED** Give an example of a binomial in the form $a\sqrt{b} + c\sqrt{d}$ and its conjugate. Then find their product.

Guided Practice

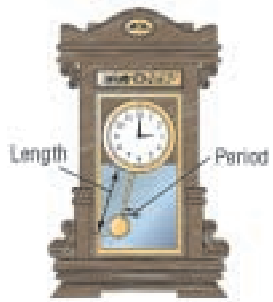
Simplify.

4. $\sqrt{20}$
5. $\sqrt{2} \cdot \sqrt{8}$
6. $3\sqrt{10} \cdot 4\sqrt{10}$
7. $\sqrt{54a^2b^2}$
8. $\sqrt{60x^5y^6}$
9. $\dfrac{4}{\sqrt{6}}$
10. $\sqrt{\dfrac{3}{10}}$
11. $\dfrac{8}{3 - \sqrt{2}}$
12. $\dfrac{2\sqrt{5}}{-4 + \sqrt{8}}$

Applications

13. **GEOMETRY** A square has sides each measuring $2\sqrt{7}$ feet. Determine the area of the square.

14. **PHYSICS** The period of a pendulum is the time required for it to make one complete swing back and forth. The formula of the period P of a pendulum is $P = 2\pi\sqrt{\frac{\ell}{32}}$, where ℓ is the length of the pendulum in feet. If a pendulum in a clock tower is 8 feet long, find the period. Use 3.14 for π.



Practice and Apply

Homework Help

For Exercises	See Examples
15–18, 41, 44–46	1
19–22, 39, 40, 48, 49	2
23–26	3
27–32, 42, 43, 47	4
33–38	5

Extra Practice

See page 844.

Simplify.

15. $\sqrt{18}$
16. $\sqrt{24}$
17. $\sqrt{80}$
18. $\sqrt{75}$
19. $\sqrt{5} \cdot \sqrt{6}$
20. $\sqrt{3} \cdot \sqrt{8}$
21. $7\sqrt{30} \cdot 2\sqrt{6}$
22. $2\sqrt{3} \cdot 5\sqrt{27}$
23. $\sqrt{40a^4}$
24. $\sqrt{50m^3n^5}$
25. $\sqrt{147x^6y^7}$
26. $\sqrt{72x^3y^4z^5}$
27. $\sqrt{\frac{2}{7}} \cdot \sqrt{\frac{7}{3}}$
28. $\sqrt{\frac{3}{5}} \cdot \sqrt{\frac{6}{4}}$
29. $\sqrt{\frac{t}{8}}$
30. $\sqrt{\frac{27}{p^2}}$
31. $\sqrt{\frac{5c^5}{4d^5}}$
32. $\frac{\sqrt{9x^5y}}{\sqrt{12x^2y^6}}$
33. $\frac{18}{6 - \sqrt{2}}$
34. $\frac{2\sqrt{5}}{-4 + \sqrt{8}}$
35. $\frac{10}{\sqrt{7} + \sqrt{2}}$
36. $\frac{2}{\sqrt{3} + \sqrt{6}}$
37. $\frac{4}{4 - 3\sqrt{3}}$
38. $\frac{3\sqrt{7}}{5\sqrt{3} + 3\sqrt{5}}$

39. **GEOMETRY** A rectangle has width $3\sqrt{5}$ centimeters and length $4\sqrt{10}$ centimeters. Find the area of the rectangle.

40. **GEOMETRY** A rectangle has length $\sqrt{\frac{a}{8}}$ meters and width $\sqrt{\frac{a}{2}}$ meters. What is the area of the rectangle?

41. **GEOMETRY** The formula for the area A of a square with side length s is $A = s^2$. Solve this equation for s, and find the side length of a square having an area of 72 square inches.

PHYSICS **For Exercises 42 and 43, use the following information.**

The formula for the kinetic energy of a moving object is $E = \frac{1}{2}mv^2$, where E is the kinetic energy in joules, m is the mass in kilograms, and v is the velocity in meters per second.

42. Solve the equation for v.

43. Find the velocity of an object whose mass is 0.6 kilogram and whose kinetic energy is 54 joules.

44. **SPACE EXPLORATION** Refer to the application at the beginning of the lesson. Find the escape velocity for the Moon in kilometers per second if $G = \frac{6.7 \times 10^{-20}\ \text{km}}{s^2\ \text{kg}}$, $M = 7.4 \times 10^{22}$ kg, and $R = 1.7 \times 10^3$ km. How does this compare to the escape velocity for Earth?

WebQuest

The speed of a roller coaster can be determined by evaluating a radical expression. Visit www.algebra1.com/webquest to continue work on your WebQuest project.

Career Choices

Insurance Investigator

Insurance investigators decide whether claims are covered by the customer's policy, assess the amount of loss, and investigate the circumstances of a claim.

Online Research For more information about a career as an insurance investigator, visit: www.algebra1.com/careers

Source: U.S. Department of Labor

INVESTIGATION **For Exercises 45–47, use the following information.**

Police officers can use the formula $s = \sqrt{30fd}$ to determine the speed s that a car was traveling in miles per hour by measuring the distance d in feet of its skid marks. In this formula, f is the coefficient of friction for the type and condition of the road.

45. Write a simplified expression for the speed if $f = 0.6$ for a wet asphalt road.

46. What is a simplified expression for the speed if $f = 0.8$ for a dry asphalt road?

47. An officer measures skid marks that are 110 feet long. Determine the speed of the car for both wet road conditions and for dry road conditions.

GEOMETRY **For Exercises 48 and 49, use the following information.**

Hero's Formula can be used to calculate the area A of a triangle given the three side lengths a, b, and c.

$$A = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where } s = \frac{1}{2}(a+b+c)$$

48. Find the value of s if the side lengths of a triangle are 13, 10, and 7 feet.

49. Determine the area of the triangle.

50. **CRITICAL THINKING** Simplify $\dfrac{1}{a - 1 + \sqrt{a}}$.

51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are radical expressions used in space exploration?

Include the following in your answer:

- an explanation of how you could determine the escape velocity for a planet and why you would need this information before you landed on the planet, and
- a comparison of the escape velocity for two astronomical bodies with the same mass, but different radii.

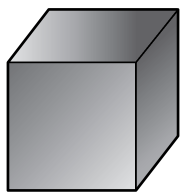
Standardized Test Practice

A B C D

52. If the cube has a surface area of $96a^2$, what is its volume?

(A) $32a^3$ (B) $48a^3$

(C) $64a^3$ (D) $96a^3$



Surface area of a cube = $6s^2$

53. If $x = 81b^2$ and $b > 0$, then $\sqrt{x} =$

(A) $-9b$. (B) $9b$.

(C) $3b\sqrt{27}$. (D) $27b\sqrt{3}$.

Graphing Calculator

WEATHER **For Exercises 54 and 55, use the following information.**

The formula $y = 91.4 - (91.4 - t)\left[0.478 + 0.301\left(\sqrt{x} - 0.02\right)\right]$ can be used to find the windchill factor. In this formula, y represents the windchill factor, t represents the air temperature in degrees Fahrenheit, and x represents the wind speed in miles per hour. Suppose the air temperature is 12°.

54. Use a graphing calculator to find the wind speed to the nearest mile per hour if it feels like −9° with the windchill factor.

55. What does it feel like to the nearest degree if the wind speed is 4 miles per hour?

www.algebra1.com/self_check_quiz

Extending the Lesson Radical expressions can be represented with fractional exponents. For example, $x^{\frac{1}{2}} = \sqrt{x}$. Using the properties of exponents, simplify each expression.

56. $x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}$

57. $\left(x^{\frac{1}{2}}\right)^4$

58. $\frac{x^{\frac{5}{2}}}{x}$

59. Simplify the expression $\frac{\sqrt{a}}{a\sqrt[3]{a}}$.

60. Solve the equation $|y^3| = \frac{1}{3\sqrt{3}}$ for y.

61. Write $\left(s^2 t^{\frac{1}{2}}\right)^8 \sqrt{s^5 t^4}$ in simplest form.

Maintain Your Skills

Mixed Review Find the next three terms in each geometric sequence. *(Lesson 10-7)*

62. 2, 6, 18, 54

63. 1, −2, 4, −8

64. 384, 192, 96, 48

65. $\frac{1}{9}, \frac{2}{3}, 4, 24$

66. $3, \frac{3}{4}, \frac{3}{16}, \frac{3}{64}$

67. 50, 10, 2, 0.4

68. **BIOLOGY** A certain type of bacteria, if left alone, doubles its number every 2 hours. If there are 1000 bacteria at a certain point in time, how many bacteria will there be 24 hours later? *(Lesson 10-6)*

69. **PHYSICS** According to Newton's Law of Cooling, the difference between the temperature of an object and its surroundings decreases in time exponentially. Suppose a cup of coffee is 95°C and it is in a room that is 20°C. The cooling of the coffee can be modeled by the equation $y = 75(0.875)^t$, where y is the temperature difference and t is the time in minutes. Find the temperature of the coffee after 15 minutes. *(Lesson 10-6)*

Factor each trinomial, if possible. If the trinomial cannot be factored using integers, write *prime*. *(Lesson 9-4)*

70. $6x^2 + 7x - 5$

71. $35x^2 - 43x + 12$

72. $5x^2 + 3x + 31$

73. $3x^2 - 6x - 105$

74. $4x^2 - 12x + 15$

75. $8x^2 - 10x + 3$

Find the solution set for each equation, given the replacement set. *(Lesson 4-4)*

76. $y = 3x + 2$; {(1, 5), (2, 6), (−2, 2), (−4, −10)}

77. $5x + 2y = 10$; {(3, 5), (2, 0), (4, 2), (1, 2.5)}

78. $3a + 2b = 11$; {(−3, 10), (4, 1), (2, 2.5), (3, −2)}

79. $5 - \frac{3}{2}x = 2y$; $\left\{(0, 1), (8, 2), \left(4, -\frac{1}{2}\right), (2, 1)\right\}$

Solve each equation. Then check your solution. *(Lesson 3-3)*

80. $40 = -5d$

81. $20.4 = 3.4y$

82. $\frac{h}{-11} = -25$

83. $-65 = \frac{r}{29}$

Getting Ready for the Next Lesson **PREREQUISITE SKILL** Find each product.

*(To review **multiplying binomials**, see Lesson 8-7.)*

84. $(x - 3)(x + 2)$

85. $(a + 2)(a + 5)$

86. $(2t + 1)(t - 6)$

87. $(4x - 3)(x + 1)$

88. $(5x + 3y)(3x - y)$

89. $(3a - 2b)(4a + 7b)$

11-2 Operations with Radical Expressions

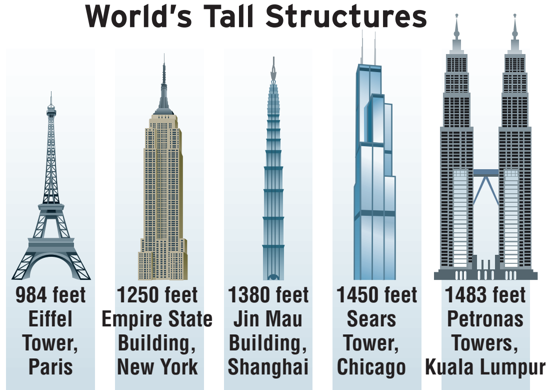
What You'll Learn

- Add and subtract radical expressions.
- Multiply radical expressions.

How can you use radical expressions to determine how far a person can see?

The formula $d = \sqrt{\frac{3h}{2}}$ represents the distance d in miles that a person h feet high can see. To determine how much farther a person can see from atop the Sears Tower than from atop the Empire State Building, we can substitute the heights of both buildings into the equation.

World's Tall Structures



ADD AND SUBTRACT RADICAL EXPRESSIONS Radical expressions in which the radicands are alike can be added or subtracted in the same way that monomials are added or subtracted.

Monomials	Radical Expressions
$2x + 7x = (2 + 7)x$	$2\sqrt{11} + 7\sqrt{11} = (2 + 7)\sqrt{11}$
$= 9x$	$= 9\sqrt{11}$
$15y - 3y = (15 - 3)y$	$15\sqrt{2} - 3\sqrt{2} = (15 - 3)\sqrt{2}$
$= 12y$	$= 12\sqrt{2}$

Notice that the Distributive Property was used to simplify each radical expression.

Example 1 Expressions with Like Radicands

Simplify each expression.

a. $\mathbf{4\sqrt{3} + 6\sqrt{3} - 5\sqrt{3}}$

$$\begin{aligned} 4\sqrt{3} + 6\sqrt{3} - 5\sqrt{3} &= (4 + 6 - 5)\sqrt{3} && \text{Distributive Property} \\ &= 5\sqrt{3} && \text{Simplify.} \end{aligned}$$

b. $\mathbf{12\sqrt{5} + 3\sqrt{7} + 6\sqrt{7} - 8\sqrt{5}}$

$$\begin{aligned} 12\sqrt{5} + 3\sqrt{7} + 6\sqrt{7} - 8\sqrt{5} &= 12\sqrt{5} - 8\sqrt{5} + 3\sqrt{7} + 6\sqrt{7} && \text{Commutative Property} \\ &= (12 - 8)\sqrt{5} + (3 + 6)\sqrt{7} && \text{Distributive Property} \\ &= 4\sqrt{5} + 9\sqrt{7} && \text{Simplify.} \end{aligned}$$

In Example 1b, $4\sqrt{5} + 9\sqrt{7}$ cannot be simplified further because the radicands are different. There are no common factors, and each radicand is in simplest form. If the radicals in a radical expression are not in simplest form, simplify them first.

Example 2 Expressions with Unlike Radicands

Simplify $2\sqrt{20} + 3\sqrt{45} + \sqrt{180}$.

$$\begin{aligned}
2\sqrt{20} + 3\sqrt{45} + \sqrt{180} &= 2\sqrt{2^2 \cdot 5} + 3\sqrt{3^2 \cdot 5} + \sqrt{6^2 \cdot 5} \\
&= 2\left(\sqrt{2^2} \cdot \sqrt{5}\right) + 3\left(\sqrt{3^2} \cdot \sqrt{5}\right) + \sqrt{6^2} \cdot \sqrt{5} \\
&= 2\left(2\sqrt{5}\right) + 3\left(3\sqrt{5}\right) + 6\sqrt{5} \\
&= 4\sqrt{5} + 9\sqrt{5} + 6\sqrt{5} \\
&= 19\sqrt{5}
\end{aligned}$$

The simplified form is $19\sqrt{5}$.

You can use a calculator to verify that a simplified radical expression is equivalent to the original expression. Consider Example 2. First, find a decimal approximation for the original expression.

KEYSTROKES: 2 [2nd] [√] 20 [ )] [+] 3 [2nd] [√] 45 [ )] [+] [2nd] [√] 180 [ )] [ENTER] 42.48529157

Next, find a decimal approximation for the simplified expression.

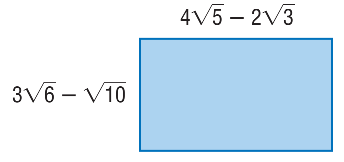
KEYSTROKES: 19 [2nd] [√] 5 [ENTER] 42.48529157

Since the approximations are equal, the expressions are equivalent.

MULTIPLY RADICAL EXPRESSIONS Multiplying two radical expressions with different radicands is similar to multiplying binomials.

Example 3 Multiply Radical Expressions

Find the area of the rectangle in simplest form.



To find the area of the rectangle multiply the measures of the length and width.

$\left(4\sqrt{5} - 2\sqrt{3}\right)\left(3\sqrt{6} - \sqrt{10}\right)$

$$\begin{aligned}
&= \overbrace{\left(4\sqrt{5}\right)\left(3\sqrt{6}\right)}^{\text{First terms}} + \overbrace{\left(4\sqrt{5}\right)\left(-\sqrt{10}\right)}^{\text{Outer terms}} + \overbrace{\left(-2\sqrt{3}\right)\left(3\sqrt{6}\right)}^{\text{Inner terms}} + \overbrace{\left(-2\sqrt{3}\right)\left(-\sqrt{10}\right)}^{\text{Last terms}} \\
&= 12\sqrt{30} - 4\sqrt{50} - 6\sqrt{18} + 2\sqrt{30} && \text{Multiply.} \\
&= 12\sqrt{30} - 4\sqrt{5^2 \cdot 2} - 6\sqrt{3^2 \cdot 2} + 2\sqrt{30} && \text{Prime factorization} \\
&= 12\sqrt{30} - 20\sqrt{2} - 18\sqrt{2} + 2\sqrt{30} && \text{Simplify.} \\
&= 14\sqrt{30} - 38\sqrt{2} && \text{Combine like terms.}
\end{aligned}$$

The area of the rectangle is $14\sqrt{30} - 38\sqrt{2}$ square units.

Study Tip

Look Back
To review the **FOIL method**, see Lesson 8-7.

Check for Understanding

Concept Check

1. **Explain** why you should simplify each radical in a radical expression before adding or subtracting.
2. **Explain** how you use the Distributive Property to simplify like radicands that are added or subtracted.
3. **OPEN ENDED** Choose values for x and y. Then find $(\sqrt{x} + \sqrt{y})^2$.

Guided Practice

Simplify each expression.

4. $4\sqrt{3} + 7\sqrt{3}$
5. $2\sqrt{6} - 7\sqrt{6}$
6. $5\sqrt{5} - 3\sqrt{20}$
7. $2\sqrt{3} + \sqrt{12}$
8. $3\sqrt{5} + 5\sqrt{6} + 3\sqrt{20}$
9. $8\sqrt{3} + \sqrt{3} + \sqrt{9}$

Find each product.

10. $\sqrt{2}(\sqrt{8} + 4\sqrt{3})$
11. $(4 + \sqrt{5})(3 + \sqrt{5})$

Applications

12. **GEOMETRY** Find the perimeter and the area of a square whose sides measure $4 + 3\sqrt{6}$ feet.

13. **ELECTRICITY** The voltage V required for a circuit is given by $V = \sqrt{PR}$, where P is the power in watts and R is the resistance in ohms. How many more volts are needed to light a 100-watt bulb than a 75-watt bulb if the resistance for both is 110 ohms?

Practice and Apply

Homework Help

For Exercises	See Examples
14–21	1
22–29	2
30–48	3

Extra Practice

See page 844.

Simplify each expression.

14. $8\sqrt{5} + 3\sqrt{5}$
15. $3\sqrt{6} + 10\sqrt{6}$
16. $2\sqrt{15} - 6\sqrt{15} - 3\sqrt{15}$
17. $5\sqrt{19} + 6\sqrt{19} - 11\sqrt{19}$
18. $16\sqrt{x} + 2\sqrt{x}$
19. $3\sqrt{5b} - 4\sqrt{5b} + 11\sqrt{5b}$
20. $8\sqrt{3} - 2\sqrt{2} + 3\sqrt{2} + 5\sqrt{3}$
21. $4\sqrt{6} + \sqrt{17} - 6\sqrt{2} + 4\sqrt{17}$
22. $\sqrt{18} + \sqrt{12} + \sqrt{8}$
23. $\sqrt{6} + 2\sqrt{3} + \sqrt{12}$
24. $3\sqrt{7} - 2\sqrt{28}$
25. $2\sqrt{50} - 3\sqrt{32}$
26. $\sqrt{2} + \sqrt{\frac{1}{2}}$
27. $\sqrt{10} - \sqrt{\frac{2}{5}}$
28. $3\sqrt{3} - \sqrt{45} + 3\sqrt{\frac{1}{3}}$
29. $6\sqrt{\frac{7}{4}} + 3\sqrt{28} - 10\sqrt{\frac{1}{7}}$

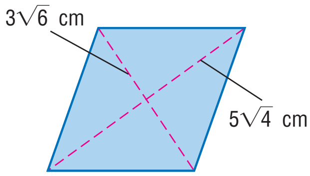
Find each product.

30. $\sqrt{6}(\sqrt{3} + 5\sqrt{2})$
31. $\sqrt{5}(2\sqrt{10} + 3\sqrt{2})$
32. $(3 + \sqrt{5})(3 - \sqrt{5})$
33. $(7 - \sqrt{10})^2$
34. $(\sqrt{6} + \sqrt{8})(\sqrt{24} + \sqrt{2})$
35. $(\sqrt{5} - \sqrt{2})(\sqrt{14} + \sqrt{35})$
36. $(2\sqrt{10} + 3\sqrt{15})(3\sqrt{3} - 2\sqrt{2})$
37. $(5\sqrt{2} + 3\sqrt{5})(2\sqrt{10} - 3)$

38. **GEOMETRY** Find the perimeter of a rectangle whose length is $8\sqrt{7} + 4\sqrt{5}$ inches and whose width is $2\sqrt{7} - 3\sqrt{5}$ inches.

39. **GEOMETRY** The perimeter of a rectangle is $2\sqrt{3} + 4\sqrt{11} + 6$ centimeters, and its length is $2\sqrt{11} + 1$ centimeters. Find the width.

40. **GEOMETRY** A formula for the area A of a rhombus can be found using the formula $A = \frac{1}{2}d_1d_2$, where d_1 and d_2 are the lengths of the diagonals of the rhombus. What is the area of the rhombus at the right?



DISTANCE **For Exercises 41 and 42, refer to the application at the beginning of the lesson.**

41. How much farther can a person see from atop the Sears Tower than from atop the Empire State Building?

42. A person atop the Empire State Building can see approximately 4.57 miles farther than a person atop the Texas Commerce Tower in Houston. Explain how you could find the height of the Texas Commerce Tower.

Online Research **Data Update** What are the tallest buildings and towers in the world today? Visit www.algebra1.com/data_update to learn more.

More About. . .

Distance

The Sears Tower was the tallest building in the world from 1974 to 1996. You can see four states from its roof: Michigan, Indiana, Illinois, and Wisconsin.

Source: www.the-skydeck.com

ENGINEERING **For Exercises 43 and 44, use the following information.**

The equation $r = \sqrt{\frac{F}{5\pi}}$ relates the radius r of a drainpipe in inches to the flow rate F of water passing through it in gallons per minute.

43. Find the radius of a pipe that can carry 500 gallons of water per minute. Round to the nearest whole number.

44. An engineer determines that a drainpipe must be able to carry 1000 gallons of water per minute and instructs the builder to use an 8-inch radius pipe. Can the builder use two 4-inch radius pipes instead? Justify your answer.

MOTION **For Exercises 45–47, use the following information.**

The velocity of an object dropped from a certain height can be found using the formula $v = \sqrt{2gd}$, where v is the velocity in feet per second, g is the acceleration due to gravity, and d is the distance in feet the object drops.

45. Find the speed of an object that has fallen 25 feet and the speed of an object that has fallen 100 feet. Use 32 feet per second squared for g.

46. When you increased the distance by 4 times, what happened to the velocity?

47. **MAKE A CONJECTURE** Estimate the velocity of an object that has fallen 225 feet. Then use the formula to verify your answer.

48. **WATER SUPPLY** The relationship between a city's size and its capacity to supply water to its citizens can be described by the expression $1020\sqrt{P}(1 - 0.01\sqrt{P})$, where P is the population in thousands and the result is the number of gallons per minute required. If a city has a population of 55,000 people, how many gallons per minute must the city's pumping station be able to supply?

49. CRITICAL THINKING Find a counterexample to disprove the following statement.

For any numbers a and b, where $a > 0$ and $b > 0$, $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$.

50. CRITICAL THINKING Under what conditions is $(\sqrt{a+b})^2 = (\sqrt{a})^2 + (\sqrt{b})^2$ true?

51. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can you use radical expressions to determine how far a person can see?

Include the following in your answer:

- an explanation of how this information could help determine how far apart lifeguard towers should be on a beach, and
- an example of a real-life situation where a lookout position is placed at a high point above the ground.

Standardized Test Practice

52. Find the difference of $9\sqrt{7}$ and $2\sqrt{28}$.

Ⓐ $\sqrt{7}$ Ⓑ $4\sqrt{7}$

Ⓒ $5\sqrt{7}$ Ⓓ $7\sqrt{7}$

53. Simplify $\sqrt{3}(4 + \sqrt{12})^2$.

Ⓐ $4\sqrt{3} + 6$ Ⓑ $28\sqrt{3}$

Ⓒ $28 + 16\sqrt{3}$ Ⓓ $48 + 28\sqrt{3}$

Maintain Your Skills

Mixed Review

Simplify. *(Lesson 11-1)*

54. $\sqrt{40}$ **55.** $\sqrt{128}$ **56.** $-\sqrt{196x^2y^3}$

57. $\frac{\sqrt{50}}{\sqrt{8}}$ **58.** $\sqrt{\frac{225c^4d}{18c^2}}$ **59.** $\sqrt{\frac{63a}{128a^3b^2}}$

Find the *n*th term of each geometric sequence. *(Lesson 10-7)*

60. $a_1 = 4, n = 6, r = 4$ **61.** $a_1 = -7, n = 4, r = 9$ **62.** $a_1 = 2, n = 8, r = -0.8$

Solve each equation by factoring. Check your solutions. *(Lesson 9-5)*

63. $81 = 49y^2$ **64.** $q^2 - \frac{36}{121} = 0$

65. $48n^3 - 75n = 0$ **66.** $5x^3 - 80x = 240 - 15x^2$

Solve each inequality. Then check your solution. *(Lesson 6-2)*

67. $8n \geq 5$ **68.** $\frac{w}{9} < 14$ **69.** $\frac{7k}{2} > \frac{21}{10}$

70. PROBABILITY A student rolls a die three times. What is the probability that each roll is a 1? *(Lesson 2-6)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each product.** *(To review **special products**, see Lesson 8-8.)*

71. $(x - 2)^2$ **72.** $(x + 5)^2$ **73.** $(x + 6)^2$

74. $(3x - 1)^2$ **75.** $(2x - 3)^2$ **76.** $(4x + 7)^2$

11-3 Radical Equations

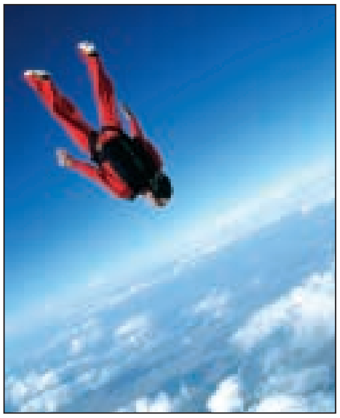
What You'll Learn

- Solve radical equations.
- Solve radical equations with extraneous solutions.

Vocabulary
- radical equation
- extraneous solution

How are radical equations used to find free-fall times?

Skydivers fall 1050 to 1480 feet every 5 seconds, reaching speeds of 120 to 150 miles per hour at *terminal velocity*. It is the highest speed they can reach and occurs when the air resistance equals the force of gravity. With no air resistance, the time t in seconds that it takes an object to fall h feet can be determined by the equation $t = \frac{\sqrt{h}}{4}$. How would you find the value of h if you are given the value of t?



RADICAL EQUATIONS Equations like $t = \frac{\sqrt{h}}{4}$ that contain radicals with variables in the radicand are called **radical equations**. To solve these equations, first isolate the radical on one side of the equation. Then square each side of the equation to eliminate the radical.

Example 1 Radical Equation with a Variable

FREE-FALL HEIGHT Two objects are dropped simultaneously. The first object reaches the ground in 2.5 seconds, and the second object reaches the ground 1.5 seconds later. From what heights were the two objects dropped?

Find the height of the first object. Replace t with 2.5 seconds.

$t = \frac{\sqrt{h}}{4}$ Original equation

$2.5 = \frac{\sqrt{h}}{4}$ Replace t with 2.5.

$10 = \sqrt{h}$ Multiply each side by 4.

$10^2 = \left(\sqrt{h}\right)^2$ Square each side.

$100 = h$ Simplify.

CHECK $t = \frac{\sqrt{h}}{4}$ Original equation

$t \stackrel{?}{=} \frac{\sqrt{100}}{4}$ $h = 100$

$t \stackrel{?}{=} \frac{10}{4}$ $\sqrt{100} = 10$

$t = 2.5$ Simplify.

The first object was dropped from 100 feet.

The time it took the second object to fall was 2.5 + 1.5 seconds or 4 seconds.

$t = \frac{\sqrt{h}}{4}$ Original equation

$4 = \frac{\sqrt{h}}{4}$ Replace t with 4.

$16 = \sqrt{h}$ Multiply each side by 4.

$16^2 = (\sqrt{h})^2$ Square each side.

$256 = h$ Simplify.

The second object was dropped from 256 feet. *Check this solution.*

Example 2 Radical Equation with an Expression

Solve $\sqrt{x+1} + 7 = 10$.

$\sqrt{x+1} + 7 = 10$ Original equation

$\sqrt{x+1} = 3$ Subtract 7 from each side.

$(\sqrt{x+1})^2 = 3^2$ Square each side.

$x + 1 = 9$ $(\sqrt{x+1})^2 = x + 1$

$x = 8$ Subtract 1 from each side.

The solution is 8. *Check this result.*

EXTRANEOUS SOLUTIONS Squaring each side of an equation sometimes produces extraneous solutions. An **extraneous solution** is a solution derived from an equation that is not a solution of the original equation. Therefore, you must check all solutions in the original equation when you solve radical equations.

Example 3 Variable on Each Side

Solve $\sqrt{x+2} = x - 4$.

$\sqrt{x+2} = x - 4$ Original equation

$(\sqrt{x+2})^2 = (x-4)^2$ Square each side.

$x + 2 = x^2 - 8x + 16$ Simplify.

$0 = x^2 - 9x + 14$ Subtract x and 2 from each side.

$0 = (x-7)(x-2)$ Factor.

$x - 7 = 0$ or $x - 2 = 0$ Zero Product Property

$x = 7$ $\quad x = 2$ Solve.

Study Tip

Look Back

To review **Zero Product Property**, see Lesson 9-2.

CHECK

$\sqrt{x+2} = x - 4$	$\sqrt{x+2} = x - 4$
$\sqrt{7+2} \stackrel{?}{=} 7 - 4$ $\quad x = 7$	$\sqrt{2+2} \stackrel{?}{=} 2 - 4$ $\quad x = 2$
$\sqrt{9} \stackrel{?}{=} 3$	$\sqrt{4} \stackrel{?}{=} -2$
$3 = 3$ ✓	$2 \neq -2$ X

Since 2 does not satisfy the original equation, 7 is the only solution.

Graphing Calculator Investigation

Solving Radical Equations

You can use a TI-83 Plus graphing calculator to solve radical equations such as $\sqrt{3x - 5} = x - 5$. Clear the Y= list. Enter the left side of the equation as Y1 $= \sqrt{3x - 5}$. Enter the right side of the equation as Y2 $= x - 5$. Press GRAPH.

Think and Discuss

1. Sketch what is shown on the screen.
2. Use the intersect feature on the CALC menu, to find the point of intersection.
3. Solve the radical equation algebraically. How does your solution compare to the solution from the graph?

Check for Understanding

Concept Check

1. **Describe** the steps needed to solve a radical equation.
2. **Explain** why it is necessary to check for extraneous solutions in radical equations.
3. **OPEN ENDED** Give an example of a radical equation. Then solve the equation for the variable.
4. **FIND THE ERROR** Alex and Victor are solving $-\sqrt{x - 5} = -2$.

Alex

$$-\sqrt{x-5} = -2$$
$$(-\sqrt{x-5})^2 = (-2)^2$$
$$x - 5 = 4$$
$$x = 9$$

Victor

$$-\sqrt{x-5} = -2$$
$$(-\sqrt{x-5})^2 = (-2)^2$$
$$-(x-5) = 4$$
$$-x + 5 = 4$$
$$x = 1$$

Who is correct? Explain your reasoning.

Guided Practice

Solve each equation. Check your solution.

5. $\sqrt{x} = 5$
6. $\sqrt{2b} = -8$
7. $\sqrt{7x} = 7$
8. $\sqrt{-3a} = 6$
9. $\sqrt{8s} + 1 = 5$
10. $\sqrt{7x + 18} = 9$
11. $\sqrt{5x + 1} + 2 = 6$
12. $\sqrt{3x - 5} = x - 5$
13. $4 + \sqrt{x - 2} = x$

Application

OCEANS **For Exercises 14–16, use the following information.**
Tsunamis, or large tidal waves, are generated by undersea earthquakes in the Pacific Ocean. The speed of the tsunami in meters per second is $s = 3.1\sqrt{d}$, where d is the depth of the ocean in meters.

14. Find the speed of the tsunami if the depth of the water is 10 meters.
15. Find the depth of the water if a tsunami's speed is 240 meters per second.
16. A tsunami may begin as a 2-foot high wave traveling 450–500 miles per hour. It can approach a coastline as a 50-foot wave. How much speed does the wave lose if it travels from a depth of 10,000 meters to a depth of 20 meters?

Practice and Apply

Homework Help

For Exercises	See Examples
17–34	1, 2
35–47	3
48–59	1–3

Extra Practice

See page 844.

Solve each equation. Check your solution.

17. $\sqrt{a} = 10$
18. $\sqrt{-k} = 4$
19. $5\sqrt{2} = \sqrt{x}$
20. $3\sqrt{7} = \sqrt{-y}$
21. $3\sqrt{4a} - 2 = 10$
22. $3 + 5\sqrt{n} = 18$
23. $\sqrt{x + 3} = -5$
24. $\sqrt{x - 5} = 2\sqrt{6}$
25. $\sqrt{3x + 12} = 3\sqrt{3}$
26. $\sqrt{2c - 4} = 8$
27. $\sqrt{4b + 1} - 3 = 0$
28. $\sqrt{3r - 5} + 7 = 3$
29. $\sqrt{\frac{4x}{5}} - 9 = 3$
30. $5\sqrt{\frac{4t}{3}} - 2 = 0$
31. $\sqrt{x^2 + 9x + 14} = x + 4$
32. $y + 2 = \sqrt{y^2 + 5y + 4}$

33. The square root of the sum of a number and 7 is 8. Find the number.

34. The square root of the quotient of a number and 6 is 9. Find the number.

Solve each equation. Check your solution.

35. $x = \sqrt{6 - x}$
36. $x = \sqrt{x + 20}$
37. $\sqrt{5x - 6} = x$
38. $\sqrt{28 - 3x} = x$
39. $\sqrt{x + 1} = x - 1$
40. $\sqrt{1 - 2b} = 1 + b$
41. $4 + \sqrt{m - 2} = m$
42. $\sqrt{3d - 8} = d - 2$
43. $x + \sqrt{6 - x} = 4$
44. $\sqrt{6 - 3x} = x + 16$
45. $\sqrt{2r^2 - 121} = r$
46. $\sqrt{5p^2 - 7} = 2p$

47. State whether the following equation is *sometimes*, *always*, or *never* true.
$\sqrt{(x - 5)^2} = x - 5$

AVIATION **For Exercises 48 and 49, use the following information.**
The formula $L = \sqrt{kP}$ represents the relationship between a plane's length L and the pounds P its wings can lift, where k is a constant of proportionality calculated for a plane.

48. The length of the Douglas D-558-II, called the Skyrocket, was approximately 42 feet, and its constant of proportionality was $k = 0.1669$. Calculate the maximum takeoff weight of the Skyrocket.

49. A Boeing 747 is 232 feet long and has a takeoff weight of 870,000 pounds. Determine the value of k for this plane.

More About. . .

Aviation

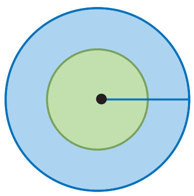
Piloted by A. Scott Crossfield on November 20, 1953, the Douglas D-558-2 Skyrocket became the first aircraft to fly faster than Mach 2, twice the speed of sound.

Source: National Air and Space Museum

GEOMETRY **For Exercises 50–53, use the figure below. The area A of a circle is equal to πr^2 where r is the radius of the circle.**

50. Write an equation for r in terms of A.

51. The area of the larger circle is 96π square meters. Find the radius.



52. The area of the smaller circle is 48π square meters. Find the radius.

53. If the area of a circle is doubled, what is the change in the radius?

PHYSICAL SCIENCE **For Exercises 54–56, use the following information.**

The formula $P = 2\pi\sqrt{\frac{\ell}{32}}$ gives the period of a pendulum of length ℓ feet. The period P is the number of seconds it takes for the pendulum to swing back and forth once.

54. Suppose we want a pendulum to complete three periods in 2 seconds. How long should the pendulum be?

55. Two clocks have pendulums of different lengths. The first clock requires 1 second for its pendulum to complete one period. The second clock requires 2 seconds for its pendulum to complete one period. How much longer is one pendulum than the other?

56. Repeat Exercise 55 if the pendulum periods are t and $2t$ seconds.

More About. . .

Physical Science

The Foucault pendulum was invented in 1851. It demonstrates the rotation of Earth. The pendulum appears to change its path during the day, but it moves in a straight line. The path under the pendulum changes because Earth is rotating beneath it.

Source: California Academy of Sciences

SOUND **For Exercises 57–59, use the following information.**

The speed of sound V near Earth's surface can be found using the equation $V = 20\sqrt{t + 273}$, where t is the surface temperature in degrees Celsius.

57. Find the temperature if the speed of sound V is 356 meters per second.

58. The speed of sound at Earth's surface is often given at 340 meters per second, but that is only accurate at a certain temperature. On what temperature is this figure based?

59. What is the speed of sound when the surface temperature is below 0°C?

60. **CRITICAL THINKING** Solve $\sqrt{h + 9} - \sqrt{h} = \sqrt{3}$.

61. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are radical equations used to find free-fall times?

Include the following in your answer:

- the time it would take a skydiver to fall 10,000 feet if he falls 1200 feet every 5 seconds and the time using the equation $t = \frac{\sqrt{h}}{4}$, with an explanation of why the two methods find different times, and
- ways that a skydiver can increase or decrease his speed.

Standardized Test Practice

QUANTITATIVE COMPARISON **In Exercises 62 and 63, compare the quantity in Column A and the quantity in Column B. Then determine whether:**

(A) the quantity in Column A is greater,

(B) the quantity in Column B is greater,

(C) both quantities are equal, or

(D) the relationship cannot be determined from the given information.

	Column A	Column B
62.	the solution of $\sqrt{x} + 3 = 6$	the solution of $\sqrt{y + 3} = 6$
63.	$(\sqrt{a - 1})^2$	$\sqrt{(a - 1)^2}$

Graphing Calculator **RADICAL EQUATIONS** Use a graphing calculator to solve each radical equation. Round to the nearest hundredth.

64. $3 + \sqrt{2x} = 7$
65. $\sqrt{3x - 8} = 5$
66. $\sqrt{x + 6} - 4 = x$
67. $\sqrt{4x + 5} = x - 7$
68. $x + \sqrt{7 - x} = 4$
69. $\sqrt{3x - 9} = 2x + 6$

Maintain Your Skills

Mixed Review

Simplify each expression. *(Lesson 11-2)*

70. $5\sqrt{6} + 12\sqrt{6}$
71. $\sqrt{12} + 6\sqrt{27}$
72. $\sqrt{18} + 5\sqrt{2} - 3\sqrt{32}$

Simplify. *(Lesson 11-1)*

73. $\sqrt{192}$
74. $\sqrt{6} \cdot \sqrt{10}$
75. $\frac{21}{\sqrt{10} + \sqrt{3}}$

Determine whether each trinomial is a perfect square trinomial. If so, factor it. *(Lesson 9-6)*

76. $d^2 + 50d + 225$
77. $4n^2 - 28n + 49$
78. $16b^2 - 56bc + 49c^2$

Find each product. *(Lesson 8-7)*

79. $(r + 3)(r - 4)$
80. $(3z + 7)(2z + 10)$
81. $(2p + 5)(3p^2 - 4p + 9)$

82. **PHYSICAL SCIENCE** A European-made hot tub is advertised to have a temperature of 35°C to 40°C, inclusive. What is the temperature range for the hot tub in degrees Fahrenheit? Use $F = \frac{9}{5}C + 32$. *(Lesson 6-4)*

Write each equation in standard form. *(Lesson 5-5)*

83. $y = 2x + \frac{3}{7}$
84. $y - 3 = -2(x - 6)$
85. $y + 2 = 7.5(x - 3)$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Evaluate $\sqrt{a^2 + b^2}$ for each value of a and b.
*(To review **evaluating expressions**, see Lesson 1-2.)*

86. $a = 3, b = 4$
87. $a = 24, b = 7$
88. $a = 1, b = 1$
89. $a = 8, b = 12$

Practice Quiz 1

Lessons 11-1 through 11-3

Simplify. *(Lesson 11-1)*

1. $\sqrt{48}$
2. $\sqrt{3} \cdot \sqrt{6}$
3. $\frac{3}{2 + \sqrt{10}}$

Simplify. *(Lesson 11-2)*

4. $6\sqrt{5} + 3\sqrt{11} + 5\sqrt{5}$
5. $2\sqrt{3} + 9\sqrt{12}$
6. $(3 - \sqrt{6})^2$

7. **GEOMETRY** Find the area of a square whose side measure is $2 + \sqrt{7}$ centimeters. *(Lesson 11-2)*

Solve each equation. Check your solution. *(Lesson 11-3)*

8. $\sqrt{15 - x} = 4$
9. $\sqrt{3x^2 - 32} = x$
10. $\sqrt{2x - 1} = 2x - 7$

Graphing Calculator Investigation

A Follow-Up of Lesson 11-3

Graphs of Radical Equations

In order for a square root to be a real number, the radicand cannot be negative. When graphing a radical equation, determine when the radicand would be negative and exclude those values from the domain.

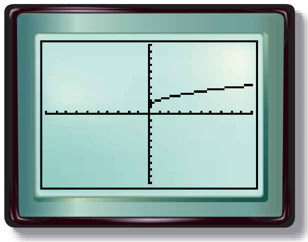
Example 1

Graph $y = \sqrt{x}$. State the domain of the graph.

Enter the equation in the Y= list.

KEYSTROKES: [Y=] [2nd] [$\sqrt{\ }$] [X,T,θ,n] [ )] [GRAPH]

From the graph, you can see that the domain of x is $\{x \mid x \geq 0\}$.



[−10, 10] scl: 1 by [−10, 10] scl: 1

Example 2

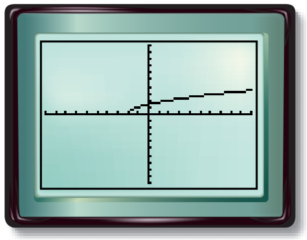
Graph $y = \sqrt{x + 4}$. State the domain of the graph.

Enter the equation in the Y= list.

KEYSTROKES: [Y=] [2nd] [$\sqrt{\ }$] [X,T,θ,n] [+] 4 [ )] [GRAPH]

The value of the radicand will be positive when $x + 4 \geq 0$, or when $x \geq -4$. So the domain of x is $\{x \mid x \geq -4\}$.

This graph looks like the graph of $y = \sqrt{x}$ shifted left 4 units.



[−10, 10] scl: 1 by [−10, 10] scl: 1

Exercises

Graph each equation and sketch the graph on your paper. State the domain of the graph. Then describe how the graph differs from the parent function $y = \sqrt{x}$.

1. $y = \sqrt{x + 1}$
2. $y = \sqrt{x} - 3$
3. $y = \sqrt{x + 2}$
4. $y = \sqrt{x - 5}$
5. $y = \sqrt{-x}$
6. $y = \sqrt{3x}$
7. $y = -\sqrt{x}$
8. $y = \sqrt{1 - x} + 6$
9. $y = \sqrt{2x + 5} - 4$

10. Is the graph of $x = y^2$ a function? Explain your reasoning.

11. Does the equation $x^2 + y^2 = 1$ determine y as a function of x? Explain.

12. Graph $y = |x| \pm \sqrt{1 - x^2}$ in the window defined by $[-2, 2]$ scl: 1 by $[-2, 2]$ scl: 1. Describe the graph.

www.algebra1.com/other_calculator_keystrokes

11-4 The Pythagorean Theorem

What You'll Learn

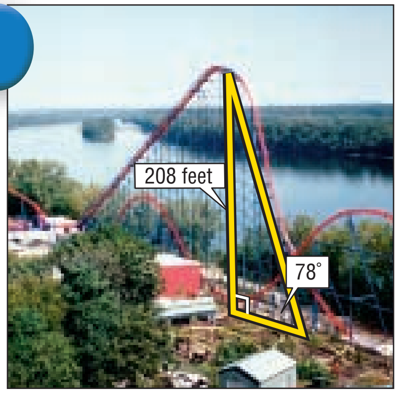
- Solve problems by using the Pythagorean Theorem.
- Determine whether a triangle is a right triangle.

Vocabulary

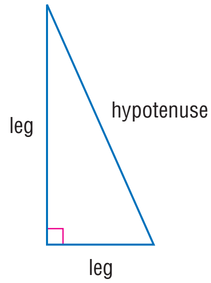
- hypotenuse
- legs
- Pythagorean triple
- corollary

How is the Pythagorean Theorem used in roller coaster design?

The roller coaster *Superman: Ride of Steel* in Agawam, Massachusetts, is one of the world's tallest roller coasters at 208 feet. It also boasts one of the world's steepest drops, measured at 78 degrees, and it reaches a maximum speed of 77 miles per hour. You can use the Pythagorean Theorem to estimate the length of the first hill.



THE PYTHAGOREAN THEOREM In a right triangle, the side opposite the right angle is called the **hypotenuse**. This side is always the longest side of a right triangle. The other two sides are called the **legs** of the triangle.



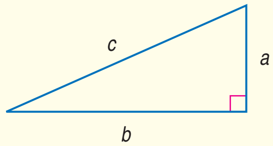
To find the length of any side of a right triangle when the lengths of the other two are known, you can use a formula developed by the Greek mathematician Pythagoras.

Study Tip

Triangles
Sides of a triangle are represented by lowercase letters *a*, *b*, and *c*.

Key Concept — The Pythagorean Theorem

- **Words** If a and b are the measures of the legs of a right triangle and c is the measure of the hypotenuse, then the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.
- **Symbols** $c^2 = a^2 + b^2$



Example 1 Find the Length of the Hypotenuse

Find the length of the hypotenuse of a right triangle if $a = 8$ and $b = 15$.

$c^2 = a^2 + b^2$ Pythagorean Theorem

$c^2 = 8^2 + 15^2$ $a = 8$ and $b = 15$

$c^2 = 289$ Simplify.

$c = \pm\sqrt{289}$ Take the square root of each side.

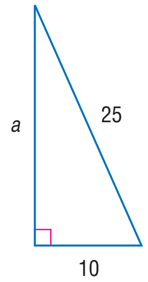
$c = \pm 17$ Disregard -17. Why?

The length of the hypotenuse is 17 units.

Example 2 Find the Length of a Side

Find the length of the missing side.

In the triangle, $c = 25$ and $b = 10$ units.



$c^2 = a^2 + b^2$	Pythagorean Theorem
$25^2 = a^2 + 10^2$	$b = 10$ and $c = 25$
$625 = a^2 + 100$	Evaluate squares.
$525 = a^2$	Subtract 100 from each side.
$\pm\sqrt{525} = a$	Use a calculator to evaluate $\sqrt{525}$.
$22.91 \approx a$	Use the positive value.

To the nearest hundredth, the length of the leg is 22.91 units.

Whole numbers that satisfy the Pythagorean Theorem are called **Pythagorean triples**. Multiples of Pythagorean triples also satisfy the Pythagorean Theorem. Some common triples are (3, 4, 5), (5, 12, 13), (8, 15, 17), and (7, 24, 25).

Standardized Test Practice

A B C D

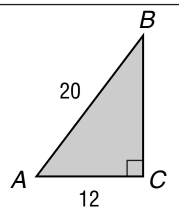
Example 3 Pythagorean Triples

Multiple-Choice Test Item

What is the area of triangle *ABC*?

(A) 96 units^2 (B) 120 units^2

(C) 160 units^2 (D) 196 units^2



Read the Test Item

The area of a triangle is $A = \frac{1}{2}bh$. In a right triangle, the legs form the base and height of the triangle. Use the measures of the hypotenuse and the base to find the height of the triangle.

Solve the Test Item

Step 1 Check to see if the measurements of this triangle are a multiple of a common Pythagorean triple. The hypotenuse is $4 \cdot 5$ units, and the leg is $4 \cdot 3$ units. This triangle is a multiple of a (3, 4, 5) triangle.

$4 \cdot 3 = 12$

$4 \cdot 4 = 16$

$4 \cdot 5 = 20$

The height of the triangle is 16 units.

Step 2 Find the area of the triangle.

$A = \frac{1}{2}bh$	Area of a triangle
$A = \frac{1}{2} \cdot 12 \cdot 16$	$b = 12$ and $h = 16$
$A = 96$	Simplify.

The area of the triangle is 96 square units. Choice A is correct.

The Princeton Review

Test-Taking Tip

Memorize the common Pythagorean triples and check for multiples such as (6, 8, 10). This will save you time when evaluating square roots.

CONTENTS

RIGHT TRIANGLES A statement that can be easily proved using a theorem is often called a **corollary**. The following corollary, based on the Pythagorean Theorem, can be used to determine whether a triangle is a right triangle.

Key Concept — Corollary to the Pythagorean Theorem

If a and b are measures of the shorter sides of a triangle, c is the measure of the longest side, and $c^2 = a^2 + b^2$, then the triangle is a right triangle. If $c^2 \neq a^2 + b^2$, then the triangle is not a right triangle.

Example 4 Check for Right Triangles

Determine whether the following side measures form right triangles.

a. 20, 21, 29

Since the measure of the longest side is 29, let $c = 29$, $a = 20$, and $b = 21$. Then determine whether $c^2 = a^2 + b^2$.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$29^2 \stackrel{?}{=} 20^2 + 21^2$	$a = 20$, $b = 21$, and $c = 29$
$841 \stackrel{?}{=} 400 + 441$	Multiply.
$841 = 841$	Add.

Since $c^2 = a^2 + b^2$, the triangle is a right triangle.

b. 8, 10, 12

Since the measure of the longest side is 12, let $c = 12$, $a = 8$, and $b = 10$. Then determine whether $c^2 = a^2 + b^2$.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$12^2 \stackrel{?}{=} 8^2 + 10^2$	$a = 8$, $b = 10$, and $c = 12$
$144 \stackrel{?}{=} 64 + 100$	Multiply.
$144 \neq 164$	Add.

Since $c^2 \neq a^2 + b^2$, the triangle is not a right triangle.

Check for Understanding

Concept Check

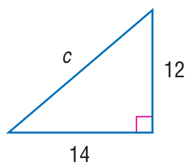
1. **OPEN ENDED** Draw a right triangle and label each side and angle. Be sure to indicate the right angle.

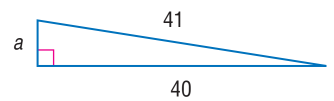
2. **Explain** how you can determine which angle is the right angle of a right triangle if you are given the lengths of the three sides.

3. **Write** an equation you could use to find the length of the diagonal d of a square with side length s.

Guided Practice

Find the length of each missing side. If necessary, round to the nearest hundredth.

4.


5.


If c is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth.

6. $a = 10, b = 24, c = ?$

7. $a = 11, c = 61, b = ?$

8. $b = 13, c = \sqrt{233}, a = ?$

9. $a = 7, b = 4, c = ?$

Determine whether the following side measures form right triangles. Justify your answer.

10. 4, 6, 9

11. 16, 30, 34

Standardized Test Practice

12. In right triangle XYZ, the length of $\overline{YZ}$ is 6, and the length of the hypotenuse is 8. Find the area of the triangle.

(A) $6\sqrt{7}$ units^2 (B) 30 units^2 (C) 40 units^2 (D) 48 units^2

Practice and Apply

Homework Help

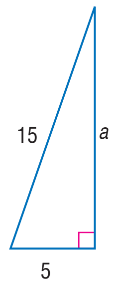
For Exercises	See Examples
13–30	1, 2
31–36	4
37–40	3

Extra Practice

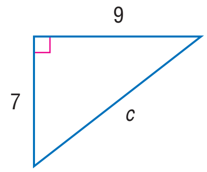
See page 845.

Find the length of each missing side. If necessary, round to the nearest hundredth.

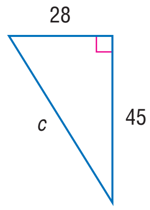
13.



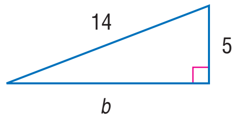
14.



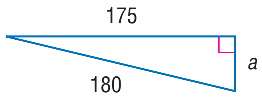
15.



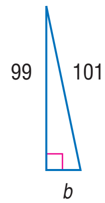
16.



17.



18.



If c is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth.

19. $a = 16, b = 63, c = ?$

20. $a = 16, c = 34, b = ?$

21. $b = 3, a = \sqrt{112}, c = ?$

22. $a = \sqrt{15}, b = \sqrt{10}, c = ?$

23. $c = 14, a = 9, b = ?$

24. $a = 6, b = 3, c = ?$

25. $b = \sqrt{77}, c = 12, a = ?$

26. $a = 4, b = \sqrt{11}, c = ?$

27. $a = \sqrt{225}, b = \sqrt{28}, c = ?$

28. $a = \sqrt{31}, c = \sqrt{155}, b = ?$

29. $a = 8x, b = 15x, c = ?$

30. $b = 3x, c = 7x, a = ?$

Determine whether the following side measures form right triangles. Justify your answer.

31. 30, 40, 50

32. 6, 12, 18

33. 24, 30, 36

34. 45, 60, 75

35. 15, $\sqrt{31}$, 16

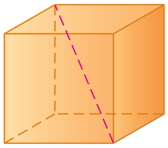
36. 4, 7, $\sqrt{65}$

Use an equation to solve each problem. If necessary, round to the nearest hundredth.

37. Find the length of a diagonal of a square if its area is 162 square feet.

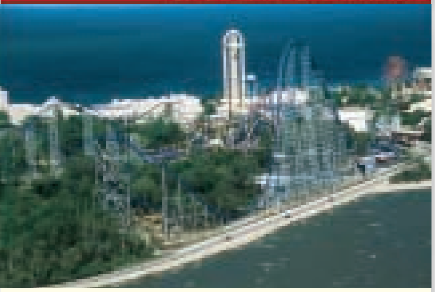
38. A right triangle has one leg that is 5 centimeters longer than the other leg. The hypotenuse is 25 centimeters long. Find the length of each leg of the triangle.

39. Find the length of the diagonal of the cube if each side of the cube is 4 inches long.



40. The ratio of the length of the hypotenuse to the length of the *shorter* leg in a right triangle is 8:5. The hypotenuse measures 144 meters. Find the length of the *longer* leg.

More About. . .



Roller Coasters

Roller Coaster Records in the U.S.

Fastest: *Millennium Force* (2000), Sandusky, Ohio; 92 mph

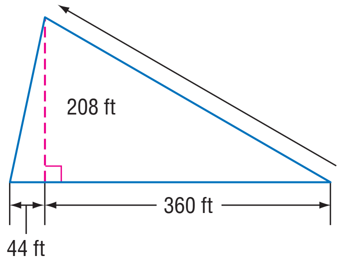
Tallest: *Millenium Force* (2000), Sandusky, Ohio; 310 feet

Longest: *California Screamin'* (2001), Anaheim, California; 6800 feet

Steepest: *Hypersonic XLC* (2001), Doswell, Virginia; 90°

Source: www.coasters2k.com

ROLLER COASTERS **For Exercises 41–43, use the following information and the figure.** Suppose a roller coaster climbs 208 feet higher than its starting point making a horizontal advance of 360 feet. When it comes down, it makes a horizontal advance of 44 feet.



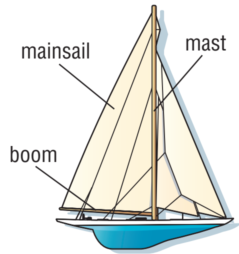
41. How far will it travel to get to the top of the ride?

42. How far will it travel on the downhill track?

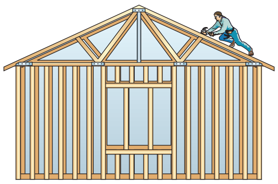
43. Compare the total horizontal advance, vertical height, and total track length.

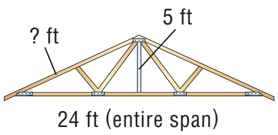
44. RESEARCH Use the Internet or other reference to find the measurements of your favorite roller coaster or a roller coaster that is at an amusement park close to you. Draw a model of the first drop. Include the height of the hill, length of the vertical drop, and steepness of the hill.

45. SAILING A sailboat's mast and boom form a right angle. The sail itself, called a *mainsail*, is in the shape of a right triangle. If the edge of the mainsail that is attached to the mast is 100 feet long and the edge of the mainsail that is attached to the boom is 60 feet long, what is the length of the longest edge of the mainsail?



ROOFING **For Exercises 46 and 47, refer to the figures below.**

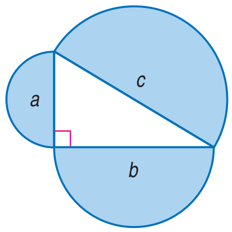




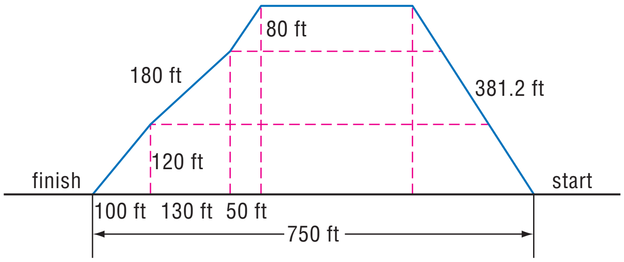
46. Determine the missing length shown in the rafter.

47. If the roof is 30 feet long and it hangs an additional 2 feet over the garage walls, how many square feet of shingles are needed for the entire garage roof?

48. CRITICAL THINKING Compare the area of the largest semicircle to the areas of the two smaller semicircles. Justify your reasoning.



49. **CRITICAL THINKING** A model of a part of a roller coaster is shown. Determine the total distance traveled from start to finish and the maximum height reached by the roller coaster.



50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

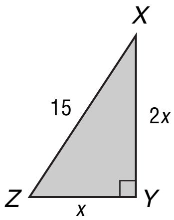
How is the Pythagorean Theorem used in roller coaster design?

Include the following in your answer:

- an explanation of how the height, speed, and steepness of a roller coaster are related, and
- a description of any limitations you can think of in the design of a new roller coaster.

Standardized Test Practice

51. Find the area of $\triangle XYZ$.



(A) $6\sqrt{5}$ units^2 (B) $18\sqrt{5}$ units^2
(C) 45 units^2 (D) 90 units^2

52. Find the perimeter of a square whose diagonal measures 10 centimeters.

(A) $10\sqrt{2}$ cm (B) $20\sqrt{2}$ cm
(C) $25\sqrt{2}$ cm (D) 80 cm

Maintain Your Skills

Mixed Review

Solve each equation. Check your solution. *(Lesson 11-3)*

53. $\sqrt{y} = 12$ 54. $3\sqrt{s} = 126$ 55. $4\sqrt{2v + 1} - 3 = 17$

Simplify each expression. *(Lesson 11-2)*

56. $\sqrt{72}$ 57. $7\sqrt{z} - 10\sqrt{z}$ 58. $\sqrt{\frac{3}{7}} + \sqrt{21}$

Simplify. Assume that no denominator is equal to zero. *(Lesson 8-2)*

59. $\frac{5^8}{5^3}$ 60. d^{-7} 61. $\frac{-26a^4b^7c^{-5}}{-13a^2b^4c^3}$

62. **AVIATION** Flying with the wind, a plane travels 300 miles in 40 minutes. Flying against the wind, it travels 300 miles in 45 minutes. Find the air speed of the plane. *(Lesson 7-4)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify each expression.**
*(To review **simplifying radical expressions**, see Lesson 11-1.)*

63. $\sqrt{(6-3)^2 + (8-4)^2}$ 64. $\sqrt{(10-4)^2 + (13-5)^2}$
65. $\sqrt{(5-3)^2 + (2-9)^2}$ 66. $\sqrt{(-9-5)^2 + (7-3)^2}$
67. $\sqrt{(-4-5)^2 + (-4-3)^2}$ 68. $\sqrt{(20-5)^2 + (-2-6)^2}$

11-5 The Distance Formula

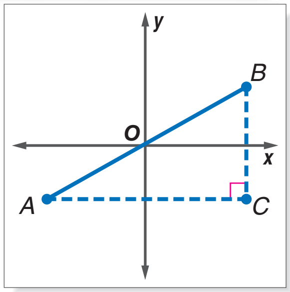
What You'll Learn

- Find the distance between two points on the coordinate plane.
- Find a point that is a given distance from a second point in a plane.

How can the distance between two points be determined?

Consider two points A and B in the coordinate plane. Notice that a right triangle can be formed by drawing lines parallel to the axes through the points at A and B. These lines intersect at C forming a right angle. The hypotenuse of this triangle is the distance between A and B. You can determine the length of the legs of this triangle and use the Pythagorean Theorem to find the distance between the two points. Notice that AC is the difference of the y-coordinates, and BC is the difference of the x-coordinates.

So, $(AB)^2 = (AC)^2 + (BC)^2$, and $AB = \sqrt{(AC)^2 + (BC)^2}$.



Study Tip

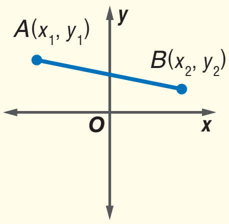
Reading Math

AC is the measure of $\overline{AC}$ and BC is the measure of $\overline{BC}$.

THE DISTANCE FORMULA You can find the distance between any two points in the coordinate plane using a similar process. The result is called the **Distance Formula**.

Key Concept — *The Distance Formula*

- **Words** The distance d between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- **Model**



Example 1 — *Distance Between Two Points*

Find the distance between the points at (2, 3) and (−4, 6).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ — Distance Formula

$= \sqrt{(-4 - 2)^2 + (6 - 3)^2}$ — $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (-4, 6)$

$= \sqrt{(-6)^2 + 3^2}$ — Simplify.

$= \sqrt{45}$ — Evaluate squares and simplify.

$= 3\sqrt{5}$ or about 6.71 units

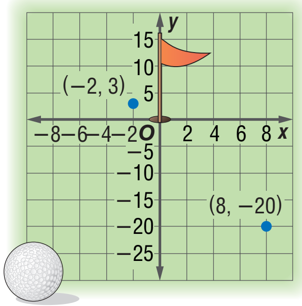
CONTENTS

Example 2 Use the Distance Formula

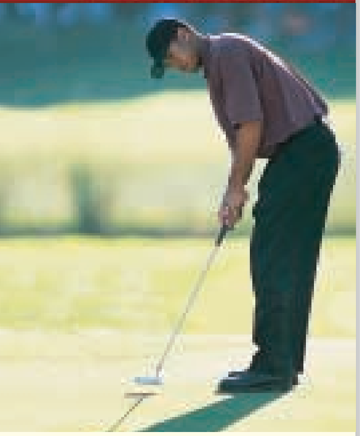
GOLF **Tracy hits a golf ball that lands 20 feet short and 8 feet to the right of the cup. On her first putt, the ball lands 2 feet to the left and 3 feet beyond the cup. Assuming that the ball traveled in a straight line, how far did the ball travel on her first putt?**

Draw a model of the situation on a coordinate grid. If the cup is at (0, 0), then the location of the ball after the first hit is (8, −20). The location of the ball after the first putt is (−2, 3). Use the Distance Formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$= \sqrt{(-2 - 8)^2 + [3 - (-20)]^2} \quad (x_1, y_1) = (8, -20), (x_2, y_2) = (-2, 3)$$
$$= \sqrt{(-10)^2 + 23^2} \quad \text{Simplify.}$$
$$= \sqrt{629} \text{ or about 25 feet}$$



More About...



Golf

There are four major tournaments that make up the "grand slam" of golf: Masters, U.S. Open, British Open, and PGA Championship. In 2000, Tiger Woods became the youngest player to win the four major events (called a career grand slam) at age 24.

Source: PGA

FIND COORDINATES Suppose you know the coordinates of a point, one coordinate of another point, and the distance between the two points. You can use the Distance Formula to find the missing coordinate.

Example 3 Find a Missing Coordinate

Find the value of *a* if the distance between the points at (7, 5) and (a, −3) is 10 units.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$
$$10 = \sqrt{(a - 7)^2 + (-3 - 5)^2} \quad \text{Let } x_2 = a, x_1 = 7, y_2 = -3, y_1 = 5,$$
$$10 = \sqrt{(a - 7)^2 + (-8)^2} \quad \text{and } d = 10.$$
$$10 = \sqrt{a^2 - 14a + 49 + 64} \quad \text{Evaluate squares.}$$
$$10 = \sqrt{a^2 - 14a + 113} \quad \text{Simplify.}$$
$$10^2 = \left(\sqrt{a^2 - 14a + 113}\right)^2 \quad \text{Square each side.}$$
$$100 = a^2 - 14a + 113 \quad \text{Simplify.}$$
$$0 = a^2 - 14a + 13 \quad \text{Subtract 100 from each side.}$$
$$0 = (a - 1)(a - 13) \quad \text{Factor.}$$
$$a - 1 = 0 \quad \text{or} \quad a - 13 = 0 \quad \text{Zero Product Property}$$
$$a = 1 \qquad a = 13$$

The value of a is 1 or 13.

Check for Understanding

Concept Check

1. **Explain** why the value calculated under the radical sign in the Distance Formula will never be negative.
2. **OPEN ENDED** Plot two ordered pairs and find the distance between their graphs. Does it matter which ordered pair is first when using the Distance Formula? Explain.
3. **Explain** why there are two values for a in Example 3. Draw a diagram to support your answer.

Guided Practice

Find the distance between each pair of points whose coordinates are given. Express in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary.

4. $(5, -1), (11, 7)$

5. $(3, 7), (-2, -5)$

6. $(2, 2), (5, -1)$

7. $(-3, -5), (-6, -4)$

Find the possible values of a if the points with the given coordinates are the indicated distance apart.

8. $(3, -1), (a, 7); d = 10$

9. $(10, a), (1, -6); d = \sqrt{145}$

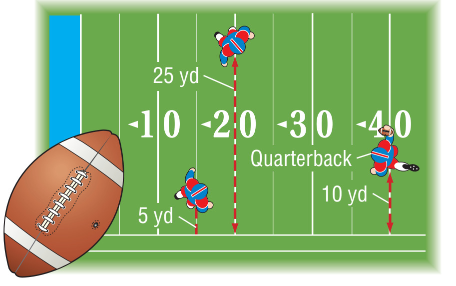
Applications

10. GEOMETRY An isosceles triangle has two sides of equal length. Determine whether triangle ABC with vertices $A(-3, 4)$, $B(5, 2)$, and $C(-1, -5)$ is an isosceles triangle.

FOOTBALL **For Exercises 11 and 12, use the information at the right.**

11. A quarterback can throw the football to one of the two receivers. Find the distance from the quarterback to each receiver.

12. What is the distance between the two receivers?



Practice and Apply

Homework Help

For Exercises	See Examples
13–26, 33, 34	1
27–32, 35, 36	3
37–42	2

Extra Practice

See page 845.

Find the distance between each pair of points whose coordinates are given. Express in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary.

13. $(12, 3), (-8, 3)$

14. $(0, 0), (5, 12)$

15. $(6, 8), (3, 4)$

16. $(-4, 2), (4, 17)$

17. $(-3, 8), (5, 4)$

18. $(9, -2), (3, -6)$

19. $(-8, -4), (-3, -8)$

20. $(2, 7), (10, -4)$

21. $(4, 2), \left(6, -\frac{2}{3}\right)$

22. $\left(5, \frac{1}{4}\right), (3, 4)$

23. $\left(\frac{4}{5}, -1\right), \left(2, -\frac{1}{2}\right)$

24. $\left(3, \frac{3}{7}\right), \left(4, -\frac{2}{7}\right)$

25. $\left(4\sqrt{5}, 7\right), \left(6\sqrt{5}, 1\right)$

26. $\left(5\sqrt{2}, 8\right), \left(7\sqrt{2}, 10\right)$

Find the possible values of a if the points with the given coordinates are the indicated distance apart.

27. $(4, 7), (a, 3); d = 5$

28. $(-4, a), (4, 2); d = 17$

29. $(5, a), (6, 1); d = \sqrt{10}$

30. $(a, 5), (-7, 3); d = \sqrt{29}$

31. $(6, -3), (-3, a); d = \sqrt{130}$

32. $(20, 5), (a, 9); d = \sqrt{340}$

33. Triangle ABC has vertices at $A(7, -4)$, $B(-1, 2)$, and $C(5, -6)$. Determine whether the triangle has three, two, or no sides that are equal in length.

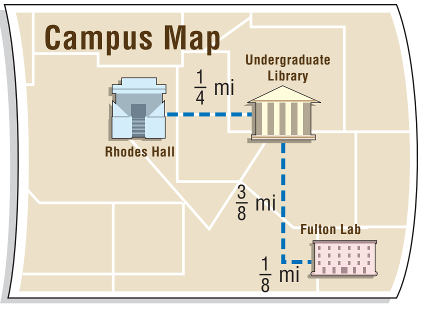
34. If the diagonals of a trapezoid have the same length, then the trapezoid is isosceles. Find the lengths of the diagonals of trapezoid $ABCD$ with vertices $A(-2, 2)$, $B(10, 6)$, $C(9, 8)$, and $D(0, 5)$ to determine if it is isosceles.

35. Triangle LMN has vertices at $L(-4, -3)$, $M(2, 5)$, and $N(-13, 10)$. If the distance from point $P(x, -2)$ to L equals the distance from P to M, what is the value of x?

36. Plot the points $Q(1, 7)$, $R(3, 1)$, $S(9, 3)$, and $T(7, d)$. Find the value of d that makes each side of $QRST$ have the same length.

37. **FREQUENT FLYERS** To determine the mileage between cities for their frequent flyer programs, some airlines superimpose a coordinate grid over the United States. An ordered pair on the grid represents the location of each airport. The units of this grid are approximately equal to 0.316 mile. So, a distance of 3 units on the grid equals an actual distance of 3(0.316) or 0.948 mile. Suppose the locations of two airports are at (132, 428) and (254, 105). Find the actual distance between these airports to the nearest mile.

COLLEGE **For Exercises 38 and 39, use the map of a college campus.**



38. Kelly has her first class in Rhodes Hall and her second class in Fulton Lab. How far does she have to walk between her first and second class?

39. She has 12 minutes between the end of her first class and the start of her second class. If she walks an average of 3 miles per hour, will she make it to her second class on time?

GEOGRAPHY **For Exercises 40–42, use the map at the left that shows part of Minnesota and Wisconsin.** A coordinate grid has been superimposed on the map with the origin at St. Paul. The grid lines are 20 miles apart. Minneapolis is at $(-7, 3)$.

40. Estimate the coordinates for Duluth, St. Cloud, Eau Claire, and Rochester.

41. Find the distance between the following pairs of cities: Minneapolis and St. Cloud, St. Paul and Rochester, Minneapolis and Eau Claire, and Duluth and St. Cloud.

42. A radio station in St. Paul has a broadcast range of 75 miles. Which cities shown on the map can receive the broadcast?

43. **CRITICAL THINKING** Plot $A(-4, 4)$, $B(-7, -3)$, and $C(4, 0)$, and connect them to form triangle ABC. Demonstrate two different ways to show whether ABC is a right triangle.

44. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can the distance between two points be determined?

Include the following in your answer:

- an explanation how the Distance Formula is derived from the Pythagorean Theorem, and
- an explanation why the Distance Formula is not needed to find the distance between points $P(-24, 18)$ and $Q(-24, 10)$.

Standardized Test Practice

45. Find the distance between points at (6, 11) and (−2, −4).

(A) 16 units (B) 17 units

(C) 18 units (D) 19 units

46. Find the perimeter of a square *ABCD* if two of the vertices are $A(3, 7)$ and $B(-3, 4)$.

(A) 12 units (B) $12\sqrt{5}$ units

(C) $9\sqrt{5}$ units (D) 45 units

Maintain Your Skills

Mixed Review

If c is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth. *(Lesson 11-4)*

47. $a = 7, b = 24, c = ?$

48. $b = 30, c = 34, a = ?$

49. $a = \sqrt{7}, c = \sqrt{16}, b = ?$

50. $a = \sqrt{13}, b = \sqrt{50}, c = ?$

Solve each equation. Check your solution. *(Lesson 11-3)*

51. $\sqrt{p - 2} + 8 = p$

52. $\sqrt{r + 5} = r - 1$

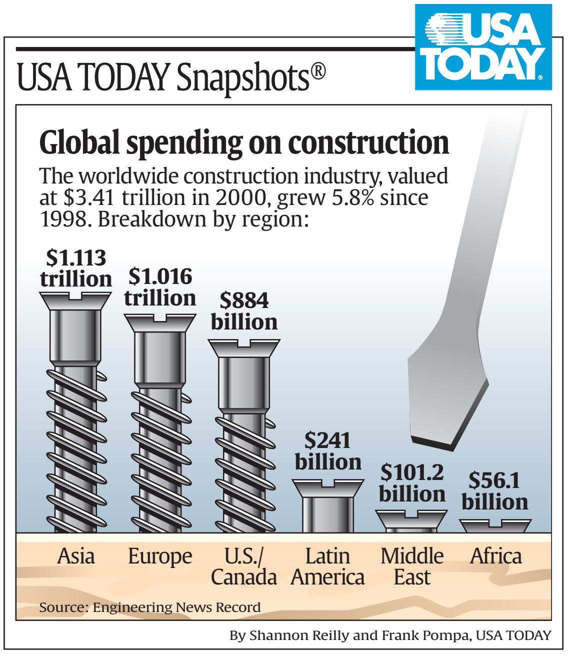
53. $\sqrt{5t^2 + 29} = 2t + 3$

COST OF DEVELOPMENT For Exercises 54–56, use the graph that shows the amount of money being spent on worldwide construction. *(Lesson 8-3)*

54. Write the value shown for each continent or region listed in standard notation.

55. Write the value shown for each continent or region in scientific notation.

56. How much more money is being spent in Asia than in Latin America?



Solve each inequality. Then check your solution and graph it on a number line. *(Lesson 6-1)*

57. $8 \leq m - 1$

58. $3 > 10 + k$

59. $3x \leq 2x - 3$

60. $v - (-4) > 6$

61. $r - 5.2 \geq 3.9$

62. $s + \frac{1}{6} \leq \frac{2}{3}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each proportion.** *(To review **proportions**, see Lesson 3-6.)*

63. $\frac{x}{4} = \frac{3}{2}$

64. $\frac{20}{x} = \frac{-5}{2}$

65. $\frac{6}{9} = \frac{8}{x}$

66. $\frac{10}{12} = \frac{x}{18}$

67. $\frac{x + 2}{7} = \frac{3}{7}$

68. $\frac{2}{3} = \frac{6}{x + 4}$

11-6 Similar Triangles

What You'll Learn

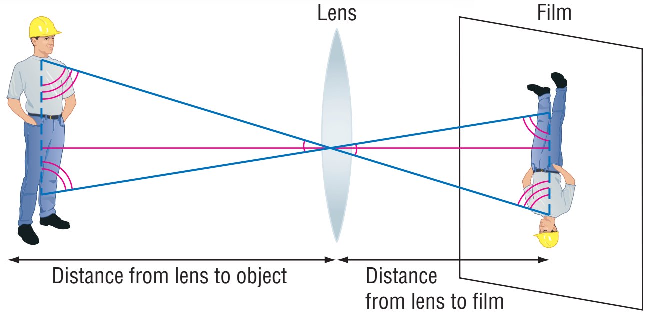
- Determine whether two triangles are similar.
- Find the unknown measures of sides of two similar triangles.

Vocabulary

- similar triangles

How are similar triangles related to photography?

When you take a picture, the image of the object being photographed is projected by the camera lens onto the film. The height of the image on the film can be related to the height of the object using similar triangles.



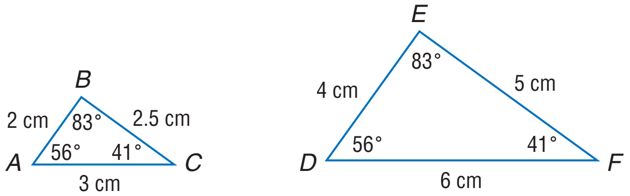
SIMILAR TRIANGLES **Similar triangles** have the same shape, but not necessarily the same size. There are two main tests for similarity.

- If the angles of one triangle and the corresponding angles of a second triangle have equal measures, then the triangles are similar.
- If the measures of the sides of two triangles form equal ratios, or are *proportional*, then the triangles are similar.

Study Tip

Reading Math
The symbol $\sim$ is read *is similar to*.

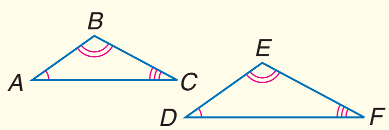
The triangles below are similar. This is written as $\triangle ABC \sim \triangle DEF$. The vertices of similar triangles are written in order to show the corresponding parts.



corresponding angles	**corresponding sides**
$\angle A$ and $\angle D$	$\overline{AB}$ and $\overline{DE} \rightarrow \frac{AB}{DE} = \frac{2}{4} = \frac{1}{2}$
$\angle B$ and $\angle E$	$\overline{BC}$ and $\overline{EF} \rightarrow \frac{BC}{EF} = \frac{2.5}{5} = \frac{1}{2}$
$\angle C$ and $\angle F$	$\overline{AC}$ and $\overline{DF} \rightarrow \frac{AC}{DF} = \frac{3}{6} = \frac{1}{2}$

Key Concept — Similar Triangles

- **Words** If two triangles are similar, then the measures of their corresponding sides are proportional, and the measures of their corresponding angles are equal.
- **Symbols** If $\triangle ABC \sim \triangle DEF$, then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.
- **Model**

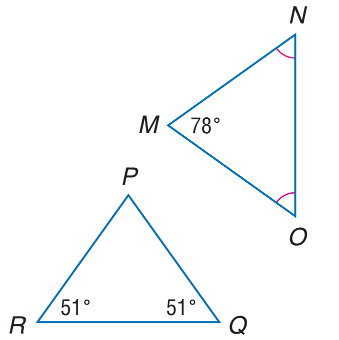


Study Tip

Reading Math
Arcs are used to show angles that have equal measures.

Example 1 Determine Whether Two Triangles Are Similar

Determine whether the pair of triangles is similar. Justify your answer.



Remember that the sum of the measures of the angles in a triangle is 180°.

The measure of $\angle P$ is $180° - (51° + 51°)$ or $78°$.

In $\triangle MNO$, $\angle N$ and $\angle O$ have the same measure.

Let x = the measure of $\angle N$ and $\angle O$.

$$x + x + 78° = 180°$$
$$2x = 102°$$
$$x = 51°$$

So $\angle N = 51°$ and $\angle O = 51°$. Since the corresponding angles have equal measures, $\triangle MNO \sim \triangle PQR$.

FIND UNKNOWN MEASURES Proportions can be used to find the measures of the sides of similar triangles when some of the measurements are known.

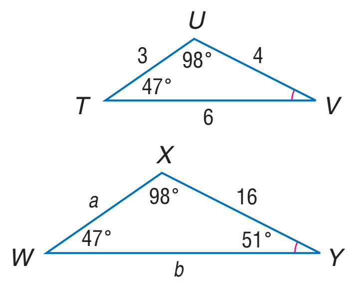
Example 2 Find Missing Measures

Study Tip

Corresponding Vertices
Always use the corresponding order of the vertices to write proportions for similar triangles.

Find the missing measures if each pair of triangles below is similar.

a. Since the corresponding angles have equal measures, $\triangle TUV \sim \triangle WXY$. The lengths of the corresponding sides are proportional.



$\frac{WX}{TU} = \frac{XY}{UV}$ Corresponding sides of similar triangles are proportional.

$\frac{a}{3} = \frac{16}{4}$ $WX = a, XY = 16, TU = 3, UV = 4$

$4a = 48$ Find the cross products.

$a = 12$ Divide each side by 4.

$\frac{WY}{TV} = \frac{XY}{UV}$ Corresponding sides of similar triangles are proportional.

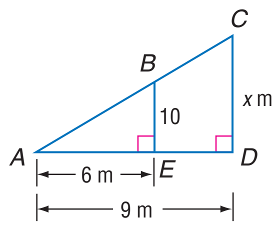
$\frac{b}{6} = \frac{16}{4}$ $WY = b, XY = 16, TV = 6, UV = 4$

$4b = 96$ Find the cross products.

$b = 24$ Divide each side by 4.

The missing measures are 12 and 24.

b. $\triangle ABE \sim \triangle ACD$



$\frac{BE}{CD} = \frac{AE}{AD}$ Corresponding sides of similar triangles are proportional.

$\frac{10}{x} = \frac{6}{9}$ $BE = 10, CD = x, AE = 6, AD = 9$

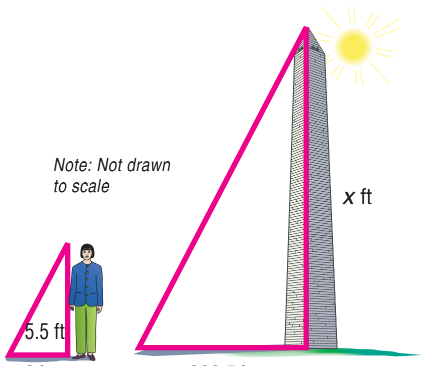
$90 = 6x$ Find the cross products.

$15 = x$ Divide each side by 6.

The missing measure is 15.

Example 3 Use Similar Triangles to Solve a Problem

SHADOWS **Jenelle is standing near the Washington Monument in Washington, D.C. The shadow of the monument is 302.5 feet, and Jenelle's shadow is 3 feet. If Jenelle is 5.5 feet tall, how tall is the monument?**



The shadows form similar triangles. Write a proportion that compares the heights of the objects and the lengths of their shadows.

Let x = the height of the monument.

$$\frac{3}{302.5} = \frac{5.5}{x}$$

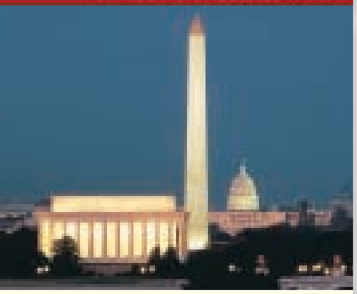
Jenelle's shadow → (numerator, left) ← Jenelle's height (numerator, right)
monument's shadow → (denominator, left) ← monument's height (denominator, right)

$3x = 1663.75$ Cross products

$x \approx 554.6$ feet Divide each side by 3.

The height of the monument is about 554.6 feet.

More About. . .



Washington Monument

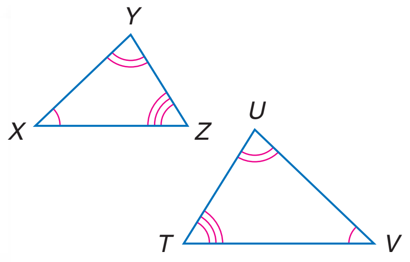
The monument has a shape of an Egyptian *obelisk*. A pyramid made of solid aluminum caps the top of the monument.

Check for Understanding

Concept Check

1. **Explain** how to determine whether two triangles are similar.
2. **OPEN ENDED** Draw a pair of similar triangles. List the corresponding angles and the corresponding sides.
3. **FIND THE ERROR** Russell and Consuela are comparing the similar triangles below to determine their corresponding parts.

Russell	Consuela
∠X = ∠T	∠X = ∠V
∠Y = ∠U	∠Y = ∠U
∠Z = ∠V	∠Z = ∠T
ΔXYZ, ΔTUV	ΔXYZ, ΔVUT

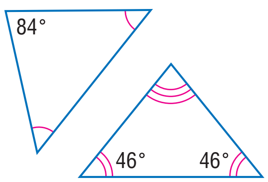


Who is correct? Explain your reasoning.

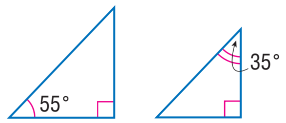
Guided Practice

Determine whether each pair of triangles is similar. Justify your answer.

4.

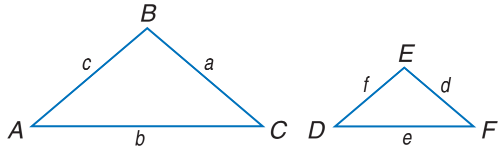


5.



For each set of measures given, find the measures of the missing sides if $\triangle ABC \sim \triangle DEF$.

6. $c = 15, d = 7, e = 9, f = 5$
7. $a = 18, c = 9, e = 10, f = 6$
8. $a = 5, d = 7, f = 6, e = 5$
9. $a = 17, b = 15, c = 10, f = 6$



Application **10. SHADOWS** If a 25-foot flagpole casts a shadow that is 10 feet long and the nearby school building casts a shadow that is 26 feet long, how high is the building?

Practice and Apply

Homework Help

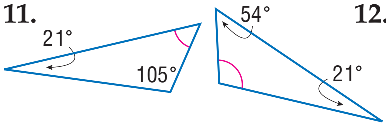
For Exercises	See Examples
11–16	1
17–24	2
25–32	3

Extra Practice

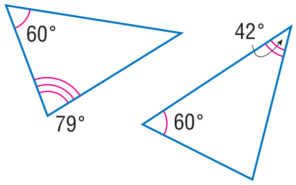
See page 845.

Determine whether each pair of triangles is similar. Justify your answer.

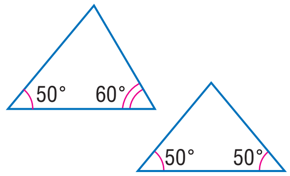
11.



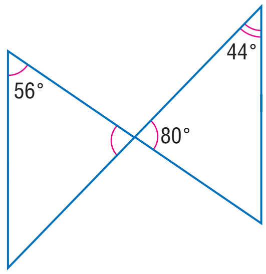
12.



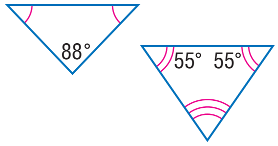
13.



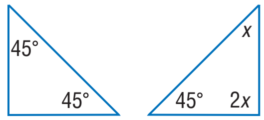
14.



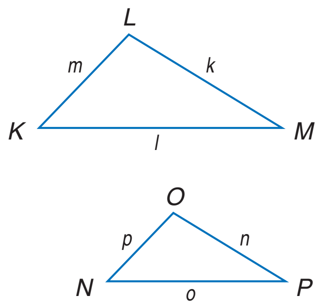
15.



16.



For each set of measures given, find the measures of the missing sides if $\triangle KLM \sim \triangle NOP$.



17. $k = 9, n = 6, o = 8, p = 4$

18. $k = 24, \ell = 30, m = 15, n = 16$

19. $m = 11, p = 6, n = 5, o = 4$

20. $k = 16, \ell = 13, m = 12, o = 7$

21. $n = 6, p = 2.5, \ell = 4, m = 1.25$

22. $p = 5, k = 10.5, \ell = 15, m = 7.5$

23. $n = 2.1, \ell = 4.5, p = 3.2, o = 3.4$

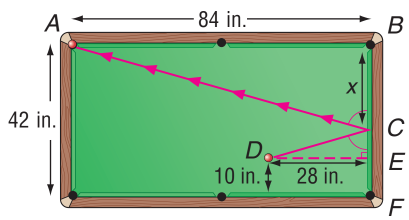
24. $m = 5, k = 12.6, o = 8.1, p = 2.5$

25. Determine whether the following statement is *sometimes*, *always*, or *never* true. *If the measures of the sides of a triangle are multiplied by 3, then the measures of the angles of the enlarged triangle will have the same measures as the angles of the original triangle.*

26. PHOTOGRAPHY Refer to the diagram of a camera at the beginning of the lesson. Suppose the image of a man who is 2 meters tall is 1.5 centimeters tall on film. If the film is 3 centimeters from the lens of the camera, how far is the man from the camera?

27. BRIDGES Truss bridges use triangles in their support beams. Mark plans to make a model of a truss bridge in the scale 1 inch = 12 feet. If the height of the triangles on the actual bridge is 40 feet, what will the height be on the model?

28. BILLIARDS Lenno is playing billiards on a table like the one shown at the right. He wants to strike the cue ball at D, bank it at C, and hit another ball at the mouth of pocket A. Use similar triangles to find where Lenno's cue ball should strike the rail.

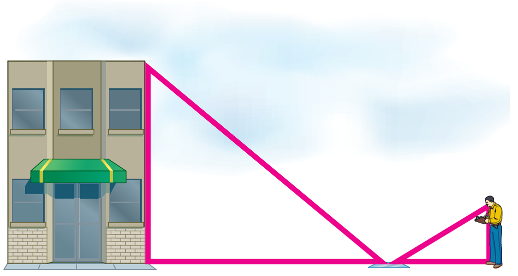


CRAFTS **For Exercises 29 and 30, use the following information.**
Melinda is working on a quilt pattern containing isosceles triangles whose sides measure 2 inches, 2 inches, and 2.5 inches.

29. She has several square pieces of material that measure 4 inches on each side. From each square piece, how many triangles with the required dimensions can she cut?

30. She wants to enlarge the pattern to make similar triangles for the center of the quilt. What is the largest similar triangle she can cut from the square material?

MIRRORS **For Exercises 31 and 32, use the diagram and the following information.**
Viho wanted to measure the height of a nearby building. He placed a mirror on the pavement at point P, 80 feet from the base of the building. He then backed away until he saw an image of the top of the building in the mirror.



31. If Viho is 6 feet tall and he is standing 9 feet from the mirror, how tall is the building?

32. What assumptions did you make in solving the problem?

CRITICAL THINKING **For Exercises 33–35, use the following information.**
The radius of one circle is twice the radius of another.

33. Are the circles similar? Explain your reasoning.

34. What is the ratio of their circumferences? Explain your reasoning.

35. What is the ratio of their areas? Explain your reasoning.

36. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

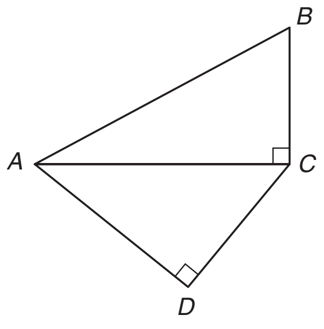
How are similar triangles related to photography?

Include the following in your answer:
- an explanation of the effect of moving a camera with a zoom lens closer to the object being photographed, and
- a description of what you could do to fit the entire image of a large object on the picture.

Standardized Test Practice

Ⓐ Ⓑ Ⓒ Ⓓ

For Exercises 37 and 38, use the figure at the right.



37. Which statement is true?

Ⓐ $\triangle ABC \sim \triangle ADC$

Ⓑ $\triangle ABC \sim \triangle ACD$

Ⓒ $\triangle ABC \sim \triangle CAD$

Ⓓ none of the above

38. Which statement is always true?

Ⓐ $AB > DC$ Ⓑ $CB > AD$

Ⓒ $AC > BC$ Ⓓ $AC = AB$

Maintain Your Skills

Mixed Review

Find the distance between each pair of points whose coordinates are given. Express answers in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary. *(Lesson 11-5)*

39. $(1, 8), (-2, 4)$

40. $(6, -3), (12, 5)$

41. $(4, 7), (3, 12)$

42. $(1, 5\sqrt{6}), (6, 7\sqrt{6})$

Determine whether the following side measures form right triangles. Justify your answer. *(Lesson 11-4)*

43. 25, 60, 65

44. 20, 25, 35

45. 49, 168, 175

46. 7, 9, 12

Arrange the terms of each polynomial so that the powers of the variable are in descending order. *(Lesson 8-4)*

47. $1 + 3x^2 - 7x$

48. $7 - 4x - 2x^2 + 5x^3$

49. $6x + 3 - 3x^2$

50. $abx^2 - bcx + 34 - x^7$

Use elimination to solve each system of equations. *(Lesson 7-3)*

51. $2x + y = 4$
$x - y = 5$

52. $3x - 2y = -13$
$2x - 5y = -5$

53. $0.6m - 0.2n = 0.9$
$0.3m = 0.45 - 0.1n$

54. $\frac{1}{3}x + \frac{1}{2}y = 8$
$\frac{1}{2}x - \frac{1}{4}y = 0$

55. AVIATION An airplane passing over Sacramento at an elevation of 37,000 feet begins its descent to land at Reno, 140 miles away. If the elevation of Reno is 4500 feet, what should be the approximate slope of descent? (*Hint*: 1 mi = 5280 ft) *(Lesson 5-1)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate if $a = 6$, $b = -5$, and $c = -1.5$.**
*(To review **evaluating expressions**, see Lesson 1-2.)*

56. $\frac{a}{c}$

57. $\frac{b}{a}$

58. $\frac{a+b}{c}$

59. $\frac{ac}{b}$

60. $\frac{b}{a+c}$

61. $\frac{c}{a+c}$

Practice Quiz 2 — Lessons 11-4 through 11-6

If c is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth. *(Lesson 11-4)*

1. $a = 14, b = 48, c = ?$

2. $a = 40, c = 41, b = ?$

3. $b = 8, c = \sqrt{84}, a = ?$

4. $a = \sqrt{5}, b = \sqrt{8}, c = ?$

Find the distance between each pair of points whose coordinates are given. *(Lesson 11-5)*

5. $(6, -12), (-3, 3)$

6. $(1, 3), (-5, 11)$

7. $(2, 5), (4, 7)$

8. $(-2, -9), (-5, 4)$

Find the measures of the missing sides if $\triangle BCA \sim \triangle EFD$. *(Lesson 11-6)*

9. $b = 10, d = 7, e = 2, f = 3$

10. $a = 12, c = 9, d = 8, e = 12$

Algebra Activity

A Preview of Lesson 11-7

Investigating Trigonometric Ratios

You can use paper triangles to investigate trigonometric ratios.

Collect the Data

Step 1 Use a ruler and grid paper to draw several right triangles whose legs are in a 7:10 ratio. Include a right triangle with legs 3.5 units and 5 units, a right triangle with legs 7 units and 10 units, another with legs 14 units and 20 units, and several more right triangles similar to these three. Label the vertices of each triangle as *A*, *B*, and *C*, where *C* is at the right angle, *B* is opposite the longest leg, and *A* is opposite the shortest leg.

Step 2 Copy the table below. Complete the first three columns by measuring the hypotenuse (side *AB*) in each right triangle you created and recording its length.

Step 3 Calculate and record the ratios in the middle two columns. Round to the nearest tenth, if necessary.

Step 4 Use a protractor to carefully measure angles *A* and *B* in each right triangle. Record the angle measures in the table.

Side Lengths			Ratios		Angle Measures		
side *BC*	side *AC*	side *AB*	*BC*:*AC*	*BC*:*AB*	angle *A*	angle *B*	angle *C*
3.5	5						90°
7	10						90°
14	20						90°
							90°
							90°
							90°

Analyze the Data

1. Examine the measures and ratios in the table. What do you notice? Write a sentence or two to describe any patterns you see.

Make a Conjecture

2. For any right triangle similar to the ones you have drawn here, what will be the value of the ratio of the length of the shortest leg to the length of the longest leg?

3. If you draw a right triangle and calculate the ratio of the length of the shortest leg to the length of the hypotenuse to be approximately 0.573, what will be the measure of the larger acute angle in the right triangle?

11-7 Trigonometric Ratios

What You'll Learn

- Define the sine, cosine, and tangent ratios.
- Use trigonometric ratios to solve right triangles.

Vocabulary

- trigonometric ratios
- sine
- cosine
- tangent
- solve a triangle
- angle of elevation
- angle of depression

How are trigonometric ratios used in surveying?

Surveyors use triangle ratios called trigonometric ratios to determine distances that cannot be measured directly.

- In 1852, British surveyors measured the altitude of the peak of Mt. Everest at 29,002 feet using these trigonometric ratios.
- In 1954, the official height became 29,028 feet, which was also calculated using surveying techniques.
- On November 11, 1999, a team using advanced technology and the Global Positioning System (GPS) satellite measured the mountain at 29,035 feet.

TRIGONOMETRIC RATIOS *Trigonometry* is an area of mathematics that involves angles and triangles. If enough information is known about a right triangle, certain ratios can be used to find the measures of the remaining parts of the triangle. **Trigonometric ratios** are ratios of the measures of two sides of a right triangle. Three common trigonometric ratios are called **sine**, **cosine**, and **tangent**.

Study Tip

Reading Math
Notice that sine, cosine, and tangent are abbreviated sin, cos, and tan respectively.

Key Concept — Trigonometric Ratios

- **Words**

$$\text{sine of } \angle A = \frac{\text{measure of leg opposite } \angle A}{\text{measure of hypotenuse}}$$

$$\text{cosine of } \angle A = \frac{\text{measure of leg adjacent to } \angle A}{\text{measure of hypotenuse}}$$

$$\text{tangent of } \angle A = \frac{\text{measure of leg opposite } \angle A}{\text{measure of leg adjacent to } \angle A}$$

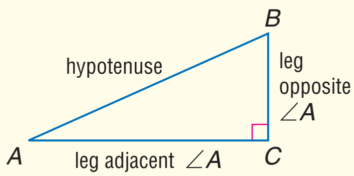
- **Symbols**

$$\sin A = \frac{BC}{AB}$$

$$\cos A = \frac{AC}{AB}$$

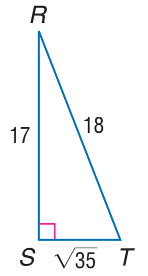
$$\tan A = \frac{BC}{AC}$$

- **Model**



Example 1 Sine, Cosine, and Tangent

Find the sine, cosine, and tangent of each acute angle of $\triangle RST$. Round to the nearest ten thousandth.



Write each ratio and substitute the measures. Use a calculator to find each value.

$$\sin R = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{\sqrt{35}}{18} \text{ or } 0.3287 \qquad \cos R = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{17}{18} \text{ or } 0.9444 \qquad \tan R = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{\sqrt{35}}{17} \text{ or } 0.3480$$

$$\sin T = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{17}{18} \text{ or } 0.9444 \qquad \cos T = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{\sqrt{35}}{18} \text{ or } 0.3287 \qquad \tan T = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{17}{\sqrt{35}} \text{ or } 2.8735$$

You can use a calculator to find the values of trigonometric functions or to find the measure of an angle. On a graphing calculator, press the trigometric function key, and then enter the value. On a nongraphing scientific calculator, enter the value, and then press the function key. In either case, be sure your calculator is in degree mode. Consider cos 50°.

Graphing Calculator	Nongraphing Scientific Calculator
KEYSTROKES: COS 50 ENTER .6427876097	**KEYSTROKES:** 50 COS .642787609

Example 2 Find the Sine of an Angle

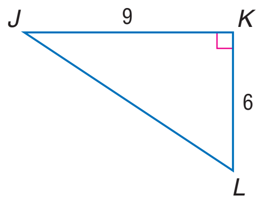
Find sin 35° to the nearest ten thousandth.

KEYSTROKES: SIN 35 ENTER .5735764364

Rounded to the nearest ten thousandth, sin 35° $\approx$ 0.5736.

Example 3 Find the Measure of an Angle

Find the measure of $\angle J$ to the nearest degree.



Since the lengths of the opposite and adjacent sides are known, use the tangent ratio.

$\tan J = \frac{\text{opposite leg}}{\text{adjacent leg}}$ Definition of tangent

$= \frac{6}{9}$ $KL = 6$ and $JK = 9$

Now use the $\boxed{\text{TAN}^{-1}}$ on a calculator to find the measure of the angle whose tangent ratio is $\frac{6}{9}$.

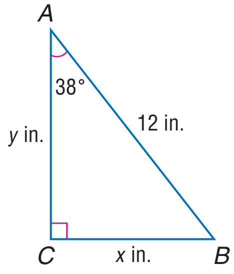
KEYSTROKES: 2nd [TAN^{-1}] 6 ÷ 9 ENTER 33.69006753

To the nearest degree, the measure of $\angle J$ is 34°.

SOLVE TRIANGLES You can find the missing measures of a right triangle if you know the measure of two sides of a triangle or the measure of one side and one acute angle. Finding all of the measures of the sides and the angles in a right triangle is called **solving the triangle**.

Example 4 Solve a Triangle

Find all of the missing measures in $\triangle ABC$.



You need to find the measures of $\angle B$, $\overline{AC}$, and $\overline{BC}$.

Step 1 Find the measure of $\angle B$. The sum of the measures of the angles in a triangle is 180.

$180° - 90° - 38° = 52°$

The measure of $\angle B$ is 52°.

Step 2 Find the value of x, which is the measure of the side opposite $\angle A$. Use the sine ratio.

$\sin 38° = \frac{x}{12}$ Definition of sine

$0.6157 \approx \frac{x}{12}$ Evaluate sin 38°.

$7.4 \approx x$ Multiply by 12.

$\overline{BC}$ is about 7.4 inches long.

Step 3 Find the value of y, which is the measure of the side adjacent to $\angle A$. Use the cosine ratio.

$\cos 38° = \frac{y}{12}$ Definition of cosine

$0.7880 \approx \frac{y}{12}$ Evaluate cos 38°.

$9.5 \approx y$ Multiply by 12.

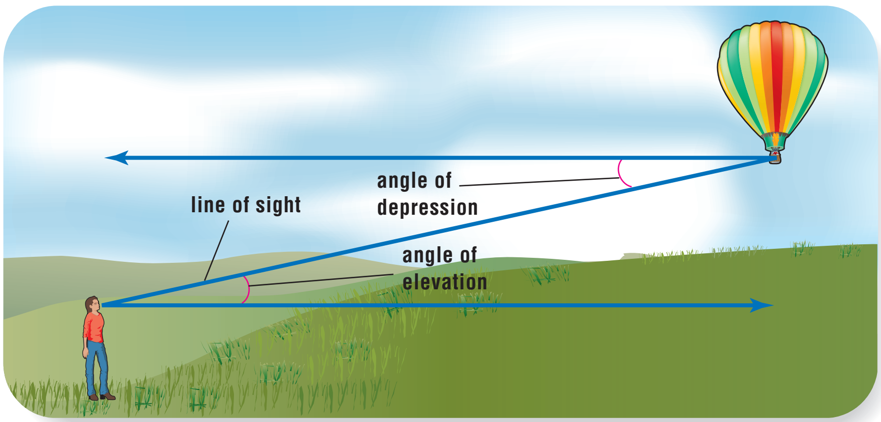
$\overline{AC}$ is about 9.5 inches long.

So, the missing measures are 52°, 7.4 in., and 9.5 in.

Study Tip

Verifying Right Triangles
You can use the Pythagorean Theorem to verify that the sides are sides of a right triangle.

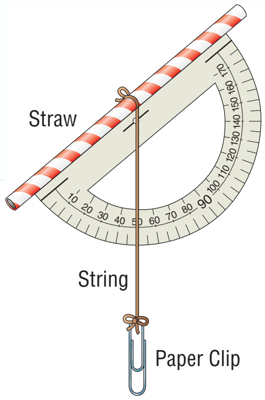
Trigonometric ratios are often used to find distances or lengths that cannot be measured directly. In these situations, you will sometimes use an angle of elevation or an angle of depression. An **angle of elevation** is formed by a horizontal line of sight and a line of sight above it. An **angle of depression** is formed by a horizontal line of sight and a line of sight below it.



Algebra Activity

Make a Hypsometer

- Tie one end of a piece of string to the middle of a straw. Tie the other end of string to a paper clip.
- Tape a protractor to the side of the straw. Make sure that the string hangs freely to create a vertical or plumb line.
- Find an object outside that is too tall to measure directly, such as a basketball hoop, a flagpole, or the school building.
- Look through the straw to the top of the object you are measuring. Find the angle measure where the string and protractor intersect. Determine the angle of elevation by subtracting this measurement from 90°.
- Measure the distance from your eye level to the ground and from your foot to the base of the object you are measuring.



Analyze

1. Make a sketch of your measurements. Use the equation

$$\tan(\text{angle of elevation}) = \frac{\text{height of object} - x}{\text{distance of object}},$$ where x represents distance from the ground to your eye level, to find the height of the object.

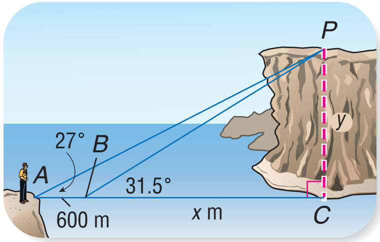
2. Why do you have to subtract the angle measurement on the hypsometer from 90° to find the angle of elevation?

3. Compare your answer with someone who measured the same object. Did your heights agree? Why or why not?

Example 5 Angle of Elevation

INDIRECT MEASUREMENT **At point A, Umeko measured the angle of elevation to point P to be 27 degrees. At another point B, which was 600 meters closer to the cliff, Umeko measured the angle of elevation to point P to be 31.5 degrees. Determine the height of the cliff.**

Explore Draw a diagram to model the situation. Two right triangles, $\triangle BPC$ and $\triangle APC$, are formed. You know the angle of elevation for each triangle. To determine the height of the cliff, find the length of $\overline{PC}$, which is shared by both triangles.



Plan Let y represent the distance from the top of the cliff P to its base C. Let x represent BC in the first triangle and let $x + 600$ represent AC.

Solve Write two equations involving the tangent ratio.

$$\tan 31.5° = \frac{y}{x} \quad \text{and} \quad \tan 27° = \frac{y}{600 + x}$$

$$x \tan 31.5° = y \qquad (600 + x)\tan 27° = y$$

Since both expressions are equal to y, use substitution to solve for x.

$$x \tan 31.5° = (600 + x) \tan 27° \quad \text{Substitute.}$$

$$x \tan 31.5° = 600 \tan 27° + x \tan 27° \quad \text{Distributive Property}$$

$$x \tan 31.5° - x \tan 27° = 600 \tan 27° \quad \text{Subtract.}$$

$$x(\tan 31.5° - \tan 27°) = 600 \tan 27° \quad \text{Isolate } x.$$

$$x = \frac{600 \tan 27°}{} \quad \text{Divide.}$$

$$x \approx 2960 \text{ feet} \quad \text{Use a calculator.}$$

Use this value for x and the equation $x \tan 31.5° = y$ to solve for y.

$$x \tan 31.5° = y \quad \text{Original equation}$$

$$2960 \tan 31.5° \approx y \quad \text{Replace } x \text{ with 2960.}$$

$$1814 \approx y \quad \text{Use a calculator.}$$

The height of the cliff is about 1814 feet.

Examine Examine the solution by finding the angles of elevation.

$\tan B = \frac{y}{x}$ | $\tan A = \frac{y}{600 + x}$

$\tan B \stackrel{?}{=} \frac{1814}{2960}$ | $\tan A \stackrel{?}{=} \frac{1814}{600 + 2960}$

$B = 31.5°$ | $A = 27°$

The solution checks.

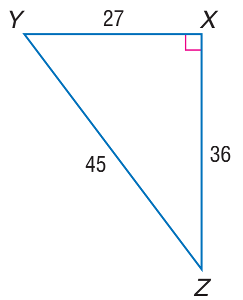
Check for Understanding

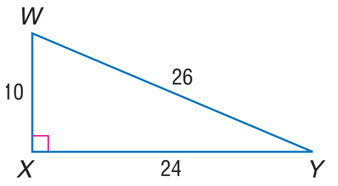
Concept Check

1. **Explain** how to determine which trigonometric ratio to use when solving for an unknown measure of a right triangle.
2. **OPEN ENDED** Draw a right triangle and label the measure of the hypotenuse and the measure of one acute angle. Then solve for the remaining measures.
3. **Compare** the measure of the angle of elevation and the measure of the angle of depression for two objects. What is the relationship between their measures?

Guided Practice

For each triangle, find sin Y, cos Y, and tan Y to the nearest ten thousandth.

4.


5.


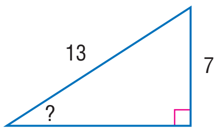
Use a calculator to find the value of each trigonometric ratio to the nearest ten thousandth.

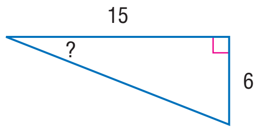
6. $\sin 60°$
7. $\cos 75°$
8. $\tan 10°$

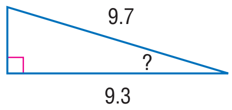
Use a calculator to find the measure of each angle to the nearest degree.

9. $\sin W = 0.9848$
10. $\cos X = 0.6157$
11. $\tan C = 0.3249$

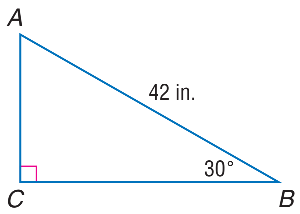
For each triangle, find the measure of the indicated angle to the nearest degree.

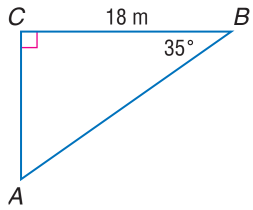
12. 

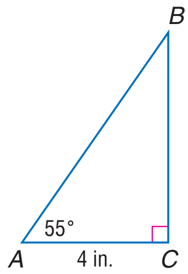
13. 

14. 

Solve each right triangle. State the side lengths to the nearest tenth and the angle measures to the nearest degree.

15. 

16. 

17. 

Application

18. DRIVING The percent grade of a road is the ratio of how much the road rises or falls in a given horizontal distance. If a road has a vertical rise of 40 feet for every 1000 feet horizontal distance, calculate the percent grade of the road and the angle of elevation the road makes with the horizontal.

Practice and Apply

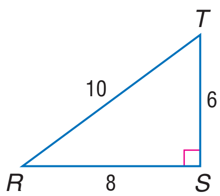
Homework Help

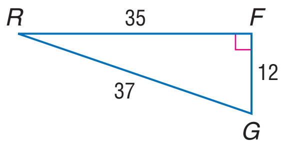
For Exercises	See Examples
19–24	1
25–33	2
34–51	3
52–60	4
61–65	5

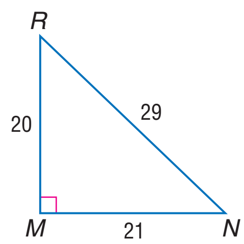
Extra Practice

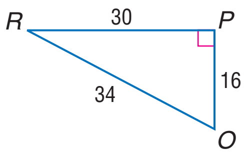
See page 846.

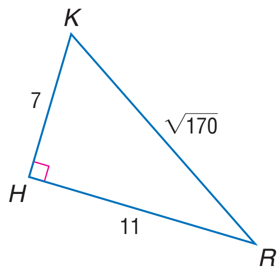
For each triangle, find sin R, cos R, and tan R to the nearest ten thousandth.

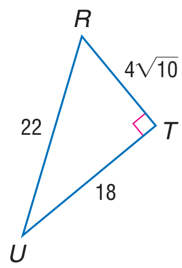
19. 

20. 

21. 

22. 

23. 

24. 

Use a calculator to find the value of each trigonometric ratio to the nearest ten thousandth.

25. sin 30° **26.** sin 80° **27.** cos 45°

28. cos 48° **29.** tan 32° **30.** tan 15°

31. tan 67° **32.** sin 53° **33.** cos 12°

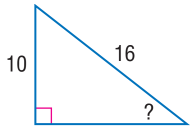
Use a calculator to find the measure of each angle to the nearest degree.

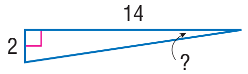
34. $\cos V = 0.5000$ **35.** $\cos Q = 0.7658$ **36.** $\sin K = 0.9781$

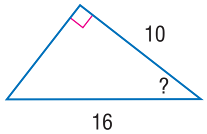
37. $\sin A = 0.8827$ **38.** $\tan S = 1.2401$ **39.** $\tan H = 0.6473$

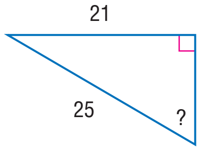
40. $\sin V = 0.3832$ **41.** $\cos M = 0.9793$ **42.** $\tan L = 3.6541$

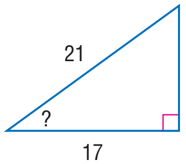
For each triangle, find the measure of the indicated angle to the nearest degree.

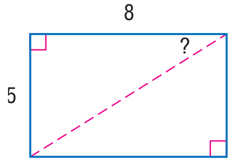
43.


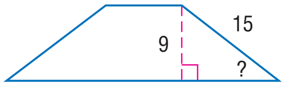
44.


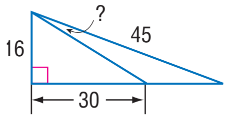
45.


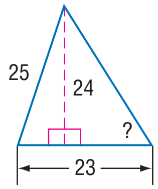
46.


47.


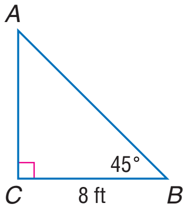
48.


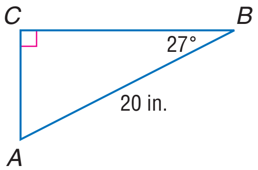
49.


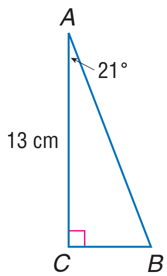
50.


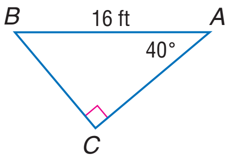
51.


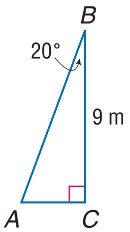
Solve each right triangle. State the side lengths to the nearest tenth and the angle measures to the nearest degree.

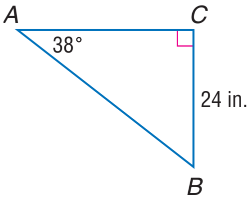
52.


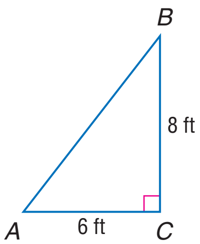
53.


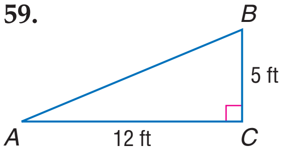
54.


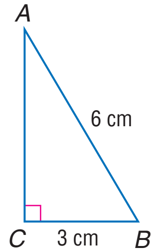
55.


56.


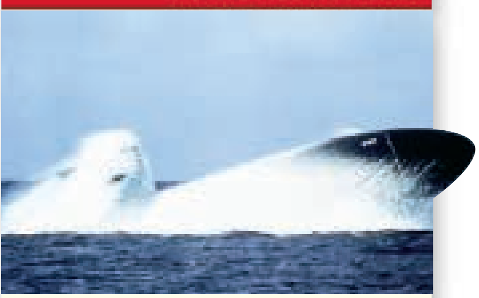
57.


58.


59.


60.


More About. . .

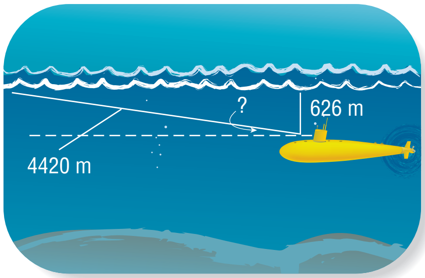


Submarines

In emergency situations, modern submarines are built to allow for a rapid surfacing, a technique called an emergency main ballast blow.

Source: www.howstuffworks.com

SUBMARINES **For Exercises 61 and 62, use the following information.**
A submarine is traveling parallel to the surface of the water 626 meters below the surface. The sub begins a constant ascent to the surface so that it will emerge on the surface after traveling 4420 meters from the point of its initial ascent.



61. What angle of ascent did the submarine make?

62. What horizontal distance did the submarine travel during its ascent?

AVIATION **For Exercises 63 and 64, use the following information.**
Germaine pilots a small plane on weekends. During a recent flight, he determined that he was flying 3000 feet parallel to the ground and that the ground distance to the start of the landing strip was 8000 feet.

63. What is Germaine's angle of depression to the start of the landing strip?

64. What is the distance between the plane in the air and the landing strip on the ground?

65. **FARMING** Leonard and Alecia are building a new feed storage system on their farm. The feed conveyor must be able to reach a range of heights. It has a length of 8 meters, and its angle of elevation can be adjusted from 20° to 5°. Under these conditions, what range of heights is possible for an opening in the building through which feed can pass?

66. **CRITICAL THINKING** An important trigonometric identity is $\sin^2 A + \cos^2 A = 1$. Use the sine and cosine ratios and the Pythagorean Theorem to prove this identity.

67. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are trigonometric ratios used in surveying?

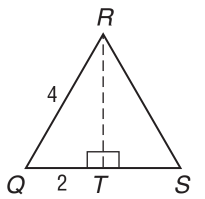
Include the following in your answer:

- an explanation of how trigonometric ratios are used to measure the height of a mountain, and
- any additional information you need to know about the point from which you are measuring in order to find the altitude of a mountain.

Standardized Test Practice

A B C D

For Exercises 68 and 69, use the figure at the right.



68. RT is equal to TS. What is RS?

(A) $2\sqrt{6}$ (B) $2\sqrt{3}$ (C) $4\sqrt{3}$ (D) $2\sqrt{2}$

69. What is the measure of $\angle Q$?

(A) 25° (B) 30° (C) 45° (D) 60°

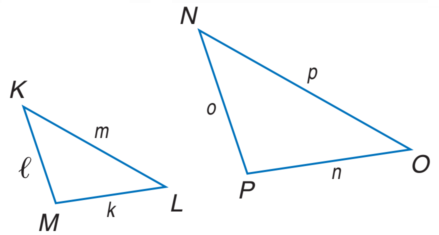
Maintain Your Skills

Mixed Review

For each set of measures given, find the measures of the missing sides if $\triangle KLM \sim \triangle NOP$. *(Lesson 11-6)*

70. $k = 5$, $\ell = 3$, $m = 6$, $n = 10$

71. $\ell = 9$, $m = 3$, $n = 12$, $p = 4.5$



Find the possible values of a if the points with the given coordinates are the indicated distance apart. *(Lesson 11-5)*

72. $(9, 28), (a, -8); d = 39$

73. $(3, a), (10, -1); d = \sqrt{65}$

Find each product. *(Lesson 8-6)*

74. $c^2(c^2 + 3c)$

75. $s(4s^2 - 9s + 12)$

76. $xy^2(2x^2 + 5xy - 7y^2)$

Use substitution to solve each system of equations. *(Lesson 7-2)*

77. $a = 3b + 2$
$4a - 7b = 23$

78. $p + q = 10$
$3p - 2q = -5$

79. $3r + 6s = 0$
$-4r - 10s = -2$

Reading Mathematics

The Language of Mathematics

The language of mathematics is a specific one, but it borrows from everyday language, scientific language, and world languages. To find a word's correct meaning, you will need to be aware of some confusing aspects of language.

Confusing Aspect	Words
Some words are used in English and in mathematics, but have distinct meanings.	factor, **leg**, prime, power, **rationalize**
Some words are used in English and in mathematics, but the mathematical meaning is more precise.	difference, even, **similar**, slope
Some words are used in science and in mathematics, but the meanings are different.	divide, **radical**, solution, variable
Some words are only used in mathematics.	decimal, **hypotenuse**, integer, quotient
Some words have more than one mathematical meaning.	base, **degree**, range, round, square
Sometimes several words come from the same root word.	polygon and polynomial, **radical** and **radicand**
Some mathematical words sound like English words.	**cosine** and cosign, **sine** and sign, sum and some
Some words are often abbreviated, but you must use the whole word when you read them.	**cos** for cosine, **sin** for sine, **tan** for tangent

Words in boldface are in this chapter.

Reading to Learn

1. How do the mathematical meanings of the following words compare to the everyday meanings?

a. factor **b.** leg **c.** rationalize

2. State two mathematical definitions for each word. Give an example for each definition.

a. degree **b.** range **c.** round

3. Each word below is shown with its root word and the root word's meaning. Find three additional words that come from the same root.

a. domain, from the root word *domus*, which means house

b. radical, from the root word *radix*, which means root

c. similar, from the root word *similis*, which means like

Chapter 11 Study Guide and Review

Vocabulary and Concept Check

angle of depression (p. 625)
angle of elevation (p. 625)
conjugate (p. 589)
corollary (p. 607)
cosine (p. 623)
Distance Formula (p. 611)
extraneous solution (p. 599)
hypotenuse (p. 605)
leg (p. 605)
Pythagorean triple (p. 606)
radical equation (p. 598)
radical expression (p. 586)
radicand (p. 586)
rationalizing the denominator (p. 588)
similar triangles (p. 616)
sine (p. 623)
solve a triangle (p. 625)
tangent (p. 623)
trigonometric ratios (p. 623)

State whether each sentence is *true* or *false*. If false, replace the underlined word, number, expression, or equation to make a true sentence.

1. The binomials $-3 + \sqrt{7}$ and $\underline{3 - \sqrt{7}}$ are conjugates.
2. In the expression $-4\sqrt{5}$, the radicand is $\underline{5}$.
3. The sine of an angle is the measure of the opposite leg divided by the measure of the <u>hypotenuse</u>.
4. The <u>longest</u> side of a right triangle is the hypotenuse.
5. After the first step in solving $\sqrt{3x + 19} = x + 3$, you would have $\underline{3x + 19 = x^2 + 9}$.
6. The two sides that form the right angle in a right triangle are called the <u>legs</u> of the triangle.
7. The expression $\underline{\dfrac{2x\sqrt{3x}}{\sqrt{6y}}}$ is in simplest radical form.
8. A triangle with sides having measures of <u>25, 20, and 15</u> is a right triangle.

Lesson-by-Lesson Review

11-1 Simplifying Radical Expressions

See pages 586–592.

Concept Summary

- A radical expression is in simplest form when no radicands have perfect square factors other than 1, no radicands contain fractions, and no radicals appear in the denominator of a fraction.

Example Simplify $\dfrac{3}{5 - \sqrt{2}}$.

$$\frac{3}{5-\sqrt{2}} = \frac{3}{5-\sqrt{2}} \cdot \frac{5+\sqrt{2}}{5+\sqrt{2}} \quad \text{Multiply by } \frac{5+\sqrt{2}}{5+\sqrt{2}} \text{ to rationalize the denominator.}$$

$$= \frac{3(5) + 3\sqrt{2}}{5^2 - (\sqrt{2})^2} \quad (a-b)(a+b) = a^2 - b^2$$

$$= \frac{15 + 3\sqrt{2}}{25 - 2} \quad (\sqrt{2})^2 = 2$$

$$= \frac{15 + 3\sqrt{2}}{23} \quad \text{Simplify.}$$

www.algebra1.com/vocabulary_review

Exercises **Simplify.** *See Examples 1–5 on pages 586–589.*

9. $\sqrt{\dfrac{60}{y^2}}$ **10.** $\sqrt{44a^2b^5}$ **11.** $(3 - 2\sqrt{12})^2$

12. $\dfrac{9}{3 + \sqrt{2}}$ **13.** $\dfrac{2\sqrt{7}}{3\sqrt{5} + 5\sqrt{3}}$ **14.** $\dfrac{\sqrt{3a^3b^4}}{\sqrt{8ab^{10}}}$

11-2 Operations with Radical Expressions

See pages 593–597.

Concept Summary

- Radical expressions with like radicands can be added or subtracted.
- Use the FOIL Method to multiply radical expressions.

Examples

1 **Simplify $\sqrt{6} - \sqrt{54} + 3\sqrt{12} + 5\sqrt{3}$.**

$\sqrt{6} - \sqrt{54} + 3\sqrt{12} + 5\sqrt{3}$

$= \sqrt{6} - \sqrt{3^2 \cdot 6} + 3\sqrt{2^2 \cdot 3} + 5\sqrt{3}$ Simplify radicands.

$= \sqrt{6} - (\sqrt{3^2} \cdot \sqrt{6}) + 3(\sqrt{2^2} \cdot \sqrt{3}) + 5\sqrt{3}$ Product Property of Square Roots

$= \sqrt{6} - 3\sqrt{6} + 3(2\sqrt{3}) + 5\sqrt{3}$ Evaluate square roots.

$= \sqrt{6} - 3\sqrt{6} + 6\sqrt{3} + 5\sqrt{3}$ Simplify.

$= -2\sqrt{6} + 11\sqrt{3}$ Add like radicands.

2 **Find $(2\sqrt{3} - \sqrt{5})(\sqrt{10} + 4\sqrt{6})$.**

$(2\sqrt{3} - \sqrt{5})(\sqrt{10} + 4\sqrt{6})$

First terms Outer terms Inner terms Last terms

$= (2\sqrt{3})(\sqrt{10}) + (2\sqrt{3})(4\sqrt{6}) + (-\sqrt{5})(\sqrt{10}) + (-\sqrt{5})(4\sqrt{6})$

$= 2\sqrt{30} + 8\sqrt{18} - \sqrt{50} - 4\sqrt{30}$ Multiply.

$= 2\sqrt{30} + 8\sqrt{3^2 \cdot 2} - \sqrt{5^2 \cdot 2} - 4\sqrt{30}$ Prime factorization

$= 2\sqrt{30} + 24\sqrt{2} - 5\sqrt{2} - 4\sqrt{30}$ Simplify.

$= -2\sqrt{30} + 19\sqrt{2}$ Combine like terms.

Exercises **Simplify each expression.** *See Examples 1 and 2 on pages 593 and 594.*

15. $2\sqrt{3} + 8\sqrt{5} - 3\sqrt{5} + 3\sqrt{3}$ **16.** $2\sqrt{6} - \sqrt{48}$

17. $4\sqrt{27} + 6\sqrt{48}$ **18.** $4\sqrt{7k} - 7\sqrt{7k} + 2\sqrt{7k}$

19. $5\sqrt{18} - 3\sqrt{112} - 3\sqrt{98}$ **20.** $\sqrt{8} + \sqrt{\dfrac{1}{8}}$

Find each product. *See Example 3 on page 594.*

21. $\sqrt{2}(3 + 3\sqrt{3})$ **22.** $\sqrt{5}(2\sqrt{5} - \sqrt{7})$

23. $(\sqrt{3} - \sqrt{2})(2\sqrt{2} + \sqrt{3})$ **24.** $(6\sqrt{5} + 2)(3\sqrt{2} + \sqrt{5})$

11-3 Radical Expressions

See pages 598–603.

Concept Summary

- Solve radical equations by isolating the radical on one side of the equation. Square each side of the equation to eliminate the radical.

Example **Solve $\sqrt{5 - 4x} - 6 = 7$.**

$\sqrt{5 - 4x} - 6 = 7$	Original equation
$\sqrt{5 - 4x} = 13$	Add 6 to each side.
$5 - 4x = 169$	Square each side.
$-4x = 164$	Subtract 5 from each side.
$x = -41$	Divide each side by -4.

Exercises **Solve each equation. Check your solution.** *See Examples 2 and 3 on page 599.*

25. $10 + 2\sqrt{b} = 0$ **26.** $\sqrt{a + 4} = 6$ **27.** $\sqrt{7x - 1} = 5$

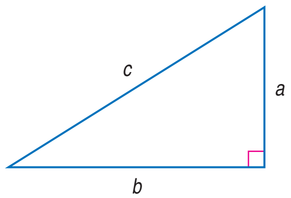
28. $\sqrt{\frac{4a}{3}} - 2 = 0$ **29.** $\sqrt{x + 4} = x - 8$ **30.** $\sqrt{3x - 14} + x = 6$

11-4 The Pythagorean Theorem

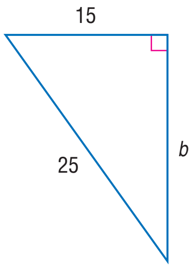
See pages 605–610.

Concept Summary

- If a and b are the measures of the legs of a right triangle and c is the measure of the hypotenuse, then $c^2 = a^2 + b^2$.
- If a and b are measures of the shorter sides of a triangle, c is the measure of the longest side, and $c^2 = a^2 + b^2$, then the triangle is a right triangle.



Example **Find the length of the missing side.**



$c^2 = a^2 + b^2$	Pythagorean Theorem
$25^2 = 15^2 + b^2$	$c = 25$ and $a = 15$
$625 = 225 + b^2$	Evaluate squares.
$400 = b^2$	Subtract 225 from each side.
$20 = b$	Take the square root of each side.

Exercises **If c is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round answers to the nearest hundredth.**
See Example 2 on page 606.

31. $a = 30, b = 16, c = ?$ **32.** $a = 6, b = 10, c = ?$ **33.** $a = 10, c = 15, b = ?$

34. $b = 4, c = 56, a = ?$ **35.** $a = 18, c = 30, b = ?$ **36.** $a = 1.2, b = 1.6, c = ?$

Determine whether the following side measures form right triangles.
See Example 4 on page 608.

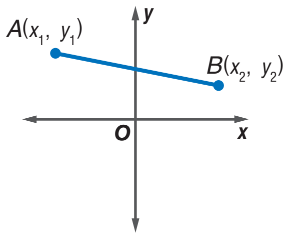
37. 9, 16, 20 **38.** 20, 21, 29 **39.** 9, 40, 41 **40.** 18, $\sqrt{24}$, 30

11-5 The Distance Formula

See pages 611–615.

Concept Summary

- The distance d between any two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



Example **Find the distance between the points with coordinates (−5, 1) and (1, 5).**

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= (1 - (-5))^2 + (5 - 1)^2$ $(x_1, y_1) = (-5, 1)$ and $(x_2, y_2) = (1, 5)$

$= \sqrt{6^2 + 4^2}$ Simplify.

$= \sqrt{36 + 16}$ Evaluate squares.

$= \sqrt{52}$ or about 7.21 units Simplify.

Exercises **Find the distance between each pair of points whose coordinates are given. Express in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary.** *See Example 1 on page 611.*

41. $(9, -2), (1, 13)$ **42.** $(4, 2), (7, 9)$ **43.** $(4, -6), (-2, 7)$

44. $(2\sqrt{5}, 9), (4\sqrt{5}, 3)$ **45.** $(4, 8), (-7, 12)$ **46.** $(-2, 6), (5, 11)$

Find the value of *a* if the points with the given coordinates are the indicated distance apart. *See Example 3 on page 612.*

47. $(-3, 2), (1, a); d = 5$ **48.** $(1, 1), (4, a); d = 5$

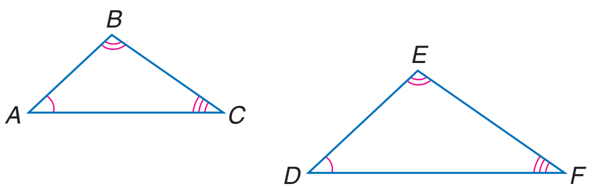
49. $(6, -2), (5, a); d = \sqrt{145}$ **50.** $(5, -2), (a, -3); d = \sqrt{170}$

11-6 Similar Triangles

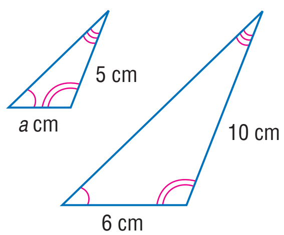
See pages 616–621.

Concept Summary

- Similar triangles have congruent corresponding angles and proportional corresponding sides.
- If $\triangle ABC \sim \triangle DEF$, then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.



Example **Find the measure of side *a* if the two triangles are similar.**



$\frac{10}{5} = \frac{6}{a}$ Corresponding sides of similar triangles are proportional.

$10a = 30$ Find the cross products.

$a = 3$ Divide each side by 10.

Chapter 11 For More ...

- Extra Practice, see pages 844–846.
- Mixed Problem Solving, see page 863.

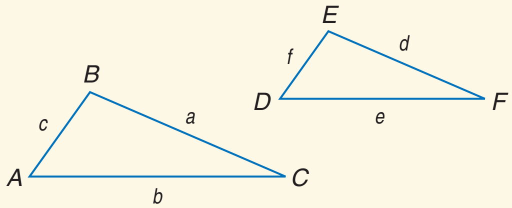
Exercises **For each set of measures given, find the measures of the remaining sides if $\triangle ABC \sim \triangle DEF$.** *See Example 2 on page 617.*

51. $c = 16, b = 12, a = 10, f = 9$

52. $a = 8, c = 10, b = 6, f = 12$

53. $c = 12, f = 9, a = 8, e = 11$

54. $b = 20, d = 7, f = 6, c = 15$



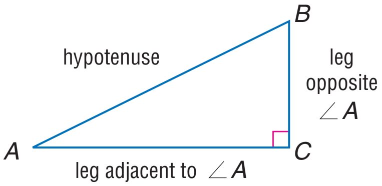
11-7 Trigonometric Ratios

See pages 623–630.

Concept Summary

Three common trigonometric ratios are sine, cosine, and tangent.

- $\sin A = \frac{BC}{AB}$
- $\cos A = \frac{AC}{AB}$
- $\tan A = \frac{BC}{AC}$

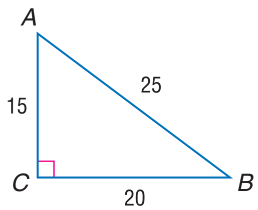


Example **Find the sine, cosine, and tangent of $\angle A$. Round to the nearest ten thousandth.**

$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}}$$
$$= \frac{20}{25} \text{ or } 0.8000$$

$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$
$$= \frac{15}{25} \text{ or } 0.6000$$

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$$
$$= \frac{20}{15} \text{ or } 1.3333$$



Exercises **For $\triangle ABC$, find each value of each trigonometric ratio to the nearest ten thousandth.** *See Example 1 on page 624.*

55. $\cos B$

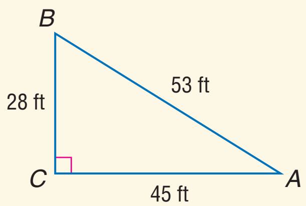
56. $\tan A$

57. $\sin B$

58. $\cos A$

59. $\tan B$

60. $\sin A$



Use a calculator to find the measure of each angle to the nearest degree.
See Example 3 on page 624.

61. $\tan M = 0.8043$ **62.** $\sin T = 0.1212$ **63.** $\cos B = 0.9781$

64. $\cos F = 0.7443$ **65.** $\sin A = 0.4540$ **66.** $\tan Q = 5.9080$

Chapter 11 Practice Test

Vocabulary and Concepts

Match each term and its definition.

1. measure of the opposite side divided by the measure of the hypotenuse
2. measure of the adjacent side divided by the measure of the hypotenuse
3. measure of the opposite side divided by the measure of the adjacent side

a. cosine
b. sine
c. tangent

Skills and Applications

Simplify.

4. $2\sqrt{27} + \sqrt{63} - 4\sqrt{3}$ **5.** $\sqrt{6} + \sqrt{\frac{2}{3}}$ **6.** $\sqrt{112x^4y^6}$

7. $\sqrt{\frac{10}{3}} \cdot \sqrt{\frac{4}{30}}$ **8.** $\sqrt{6}(4 + \sqrt{12})$ **9.** $(1 - \sqrt{3})(3 + \sqrt{2})$

Solve each equation. Check your solution.

10. $\sqrt{10x} = 20$ **11.** $\sqrt{4s} + 1 = 11$ **12.** $\sqrt{4x + 1} = 5$

13. $x = \sqrt{-6x - 8}$ **14.** $x = \sqrt{5x + 14}$ **15.** $\sqrt{4x - 3} = 6 - x$

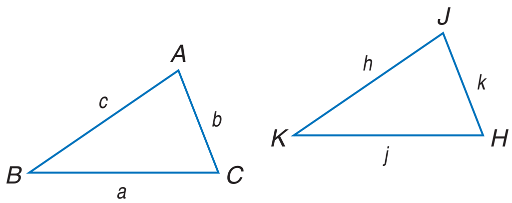
If c is the measure of the hypotenuse of a right triangle, find each missing measure. If necessary, round to the nearest hundredth.

16. $a = 8, b = 10, c = ?$ **17.** $a = 6\sqrt{2}, c = 12, b = ?$ **18.** $b = 13, c = 17, a = ?$

Find the distance between each pair of points whose coordinates are given. Express in simplest radical form and as decimal approximations rounded to the nearest hundredth if necessary.

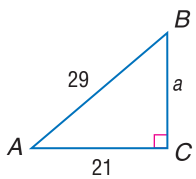
19. $(4, 7), (4, -2)$ **20.** $(-1, 1), (1, -5)$ **21.** $(-9, 2), (21, 7)$

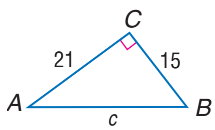
For each set of measures given, find the measures of the missing sides if $\triangle ABC \sim \triangle JKH$.

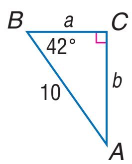


22. $c = 20, h = 15, k = 16, j = 12$
23. $c = 12, b = 13, a = 6, h = 10$
24. $k = 5, c = 6.5, b = 7.5, a = 4.5$
25. $h = 1\frac{1}{2}, c = 4\frac{1}{2}, k = 2\frac{1}{4}, a = 3$

Solve each right triangle. State the side lengths to the nearest tenth and the angle measures to the nearest degree.

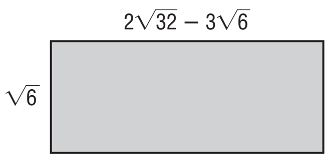
26.


27.


28.


29. **SPORTS** A hiker leaves her camp in the morning. How far is she from camp after walking 9 miles due west and then 12 miles due north?

30. **STANDARDIZED TEST PRACTICE** Find the area of the rectangle.



Ⓐ $16\sqrt{2} - 4\sqrt{6}$ units^2 Ⓑ $16\sqrt{3} - 18$ units^2
Ⓒ $32\sqrt{3} - 18$ units^2 Ⓓ $2\sqrt{32} - 18$ units^2

Chapter 11 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Which equation describes the data in the table? (Lesson 4-8)

x	−5	−2	1	4
y	11	5	−1	−7

Ⓐ $y = x - 6$ Ⓑ $y = 2x - 1$

Ⓒ $y = 2x + 1$ Ⓓ $y = -2x + 1$

2. The length of a rectangle is 6 feet more than the width. The perimeter is 92 feet. Which system of equations will determine the length in feet ℓ and the width in feet w of the rectangle? (Lesson 7-2)

Ⓐ $w = \ell + 6$, $2\ell + 2w = 92$ Ⓑ $\ell + w = 6$, $\ell w = 92$

Ⓒ $\ell = w + 6$, $2\ell + 2w = 92$ Ⓓ $\ell - w = 6$, $\ell + w = 92$

3. A highway resurfacing project and a bridge repair project will cost $2,500,000 altogether. The bridge repair project will cost $200,000 less than twice the cost of the highway resurfacing. How much will the highway resurfacing project cost? (Lesson 7-2)

Ⓐ $450,000 Ⓑ $734,000

Ⓒ $900,000 Ⓓ $1,600,000

4. If 32,800,000 is expressed in the form 3.28×10^n, what is the value of n? (Lesson 8-3)

Ⓐ 5 Ⓑ 6

Ⓒ 7 Ⓓ 8

5. What are the solutions of the equation $x^2 + 7x - 18 = 0$? (Lesson 9-4)

Ⓐ 2 or −9 Ⓑ −2 or 9

Ⓒ −2 or −9 Ⓓ 2 or 9

6. The function $g = t^2 - t$ represents the total number of games played by t teams in a sports league in which each team plays each of the other teams twice. The Metro League plays a total of 132 games. How many teams are in the league? (Lesson 9-4)

Ⓐ 11 Ⓑ 12

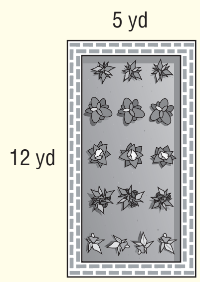
Ⓒ 22 Ⓓ 33

7. One leg of a right triangle is 4 inches longer than the other leg. The hypotenuse is 20 inches long. What is the length of the shorter leg? (Lesson 11-4)

Ⓐ 10 in. Ⓑ 12 in.

Ⓒ 16 in. Ⓓ 18 in.

8. What is the distance from one corner of the garden to the opposite corner? (Lesson 11-4)



Ⓐ 13 yards

Ⓑ 14 yards

Ⓒ 15 yards

Ⓓ 17 yards

9. How many points in the coordinate plane are equidistant from both the x- and y-axes and are 5 units from the origin? (Lesson 11-5)

Ⓐ 0 Ⓑ 1

Ⓒ 2 Ⓓ 4

The Princeton Review **Test-Taking Tip**

Questions 7, 21, and 22 Be sure that you know and understand the Pythagorean Theorem. References to right angles, the diagonal of a rectangle, or the hypotenuse of a triangle indicate that you may need to use the Pythagorean Theorem to find the answer to an item.

Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. A line is parallel to the line represented by the equation $\frac{1}{2}y + \frac{3}{2}x + 4 = 0$. What is the slope of the parallel line? (Lesson 5-6)

11. Graph the solution of the system of linear inequalities $2x - y > 2$ and $3x + 2y < -4$. (Lesson 6-6)

12. The sum of two integers is 66. The second integer is 18 more than half of the first. What are the integers? (Lesson 7-2)

13. The function $h(t) = -16t^2 + v_0 t + h_0$ describes the height in feet above the ground $h(t)$ of an object thrown vertically from a height of h_0 feet, with an initial velocity of v_0 feet per second, if there is no air friction and t is the time in seconds that it takes the ball to reach the ground. A ball is thrown upward from a 100-foot tower at a velocity of 60 feet per second. How many seconds will it take for the ball to reach the ground? (Lesson 9-5)

14. Find all values of x that satisfy the equation $x^2 - 8x + 6 = 0$. Approximate irrational numbers to the nearest hundredth. (Lesson 10-4)

15. Simplify the expression $\sqrt[3]{3\sqrt{81}}$. (Lesson 11-1)

16. Simplify the expression $\left(x^{\frac{3}{2}}\right)^{\frac{4}{3}}\left(\frac{\sqrt{x}}{x}\right)$. (Lesson 11-1)

17. The area of a rectangle is 64. The length is $\frac{x^3}{x+1}$, and the width is $\frac{x+1}{x}$. What is x? (Lesson 11-3)

Part 3 Quantative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- Ⓐ **the quantity in Column A is greater,**
- Ⓑ **the quantity in Column B is greater,**
- Ⓒ **the two quantities are equal, or**
- Ⓓ **the relationship cannot be determined from the information given.**

	Column A	Column B
18.	the value of x in $-13x - 12 = -10x + 3$	the value of y in $12y + 16 = 8y$
19.	the slope of $2x - 3y = 10$	the y-intercept of $7x + 4y = 4$
20.	the measure of the hypotenuse of a right triangle if the measures of the legs are 10 and 11	the measure of the leg of a right triangle if the measure of the other leg is 13 and the hypotenuse is $\sqrt{390}$

18. (Lesson 3-5)
19. (Lesson 5-3)
20. (Lesson 11-4)

Part 4 Open Ended

Record your answers on a sheet of paper. Show your work.

21. Haley hikes 3 miles north, 7 miles east, and then 6 miles north again. (Lesson 11-4)

a. Draw a diagram showing the direction and distance of each segment of Haley's hike. Label Haley's starting point, her ending point, and the distance, in miles, of each segment of her hike.

b. To the nearest tenth of a mile, how far (in a straight line) is Haley from her starting point?

c. How did your diagram help you to find Haley's distance from her starting point?

www.algebra1.com/standardized_test

CONTENTS